Stochastic Actor-Oriented Models
and
“Change we can believe in: Comparing longitudinal network models on consistency, interpretability and predictive power”

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Overview

General questions

1. How to conceptualize and model time?
    - Temporal ordering of social process: sequence or simultaneity?
    - *One may argue that true simultaneity never occurs in physical systems and consequently does not occur in social systems either. Yet, a distinction needs to be made between absolute time and measurable time from the point of view of the entities that cause edge formation.* Leifeld and Cranmer 2016
    - Collective action: *Theory assumes that groups collectively engage in non-action until the incentive structures are changed such that all actors have an incentive to become active at once.* Leifeld and Cranmer 2016

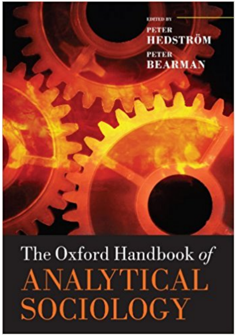
General questions

2. What is the role of prediction?
    - Compare models based on explanatory or predictive power?
    - Some models not suited for some predictions.
      Block, et al. 2017

Logic of SAOM (SIENA)

Stochastic actor-oriented models (SAOM):

- Analyze network panel data
- Insight into actor-level mechanisms that change social structure
- Goal: Explain micro-macro processes — *Analytical Sociology*
- Statistical tool: Simulation Investigation for Empirical Network Analysis (SIENA)



Hedström and Bearman 2011

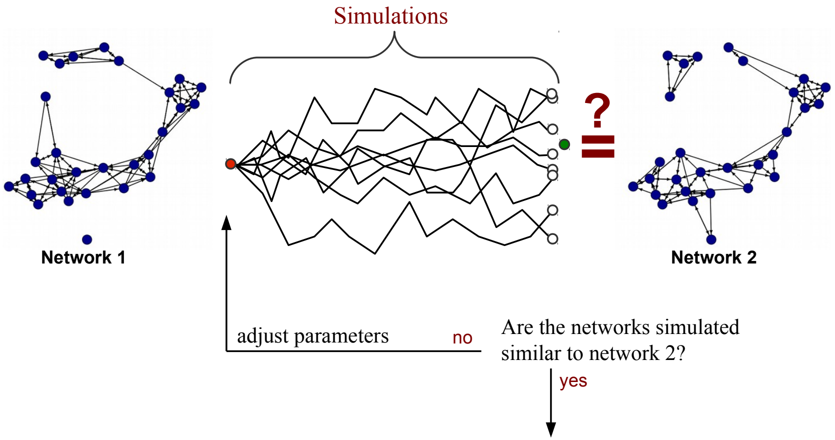
Logic of SAOM (SIENA)

SIENA findings: *Which mechanisms ("effects") are responsible for observed network states over time?*

1. Node is selected randomly ("rate of change function")
2. Node has opportunity to change state of one dyad (create, dissolve, or maintain tie/no-tie)
3. Selection based on current state of nodes' ties and the network ("objective function")
4. After change, other node selected
   - *Each sequence is a "mini-step"*
5. Process (*all mini-steps*) simulated between observed waves
6. Output checked against observed data

Logic of SAOM (SIENA)

Stadtfeld 2016:



The parameters are "good" descriptors of the social processes shaping the social network

Logic of SAOM (SIENA)

1. Selecting actor (node) with *rate-of-change function*:

$$\forall i : \quad \lambda_i(N^t) = \rho_t$$

- Random Poisson process: each node as equal probability of being chosen
- Node "wait time" can be weighted by node attributes

Logic of SAOM (SIENA)

2. *Output*: Log-odds of...

t_1	t_2	
$i \quad j$	$i \rightarrow j$	creation of tie
$i \rightarrow j$	$i \rightarrow j$	maintenance of a tie

versus

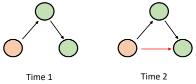
$i \rightarrow j$	$i \quad j$	termination of a tie
$i \quad j$	$i \quad j$	maintenance of a "no-tie"

Logic of SAOM (SIENA)

3. Modeling attractiveness of network states (x) for actor (i) with *objective function*:

$$f_i(x) = \sum_k \beta_k s_{ik}(x)$$

- Statistics s_{ik} are “effects” (k indexes a specific effect)
- Statistics weighted by model parameters β_k
- Weights express whether statistic is desired ($\beta_k > 0$) or averted ($\beta_k < 0$)

Effect name	Formula	Depiction ($t_1 \to t_2$)
transitive triplets	$s_i(x) = \sum_{i,j} x_{ij} x_{ih} x_{hj}$	

Logic of SAOM (SIENA)

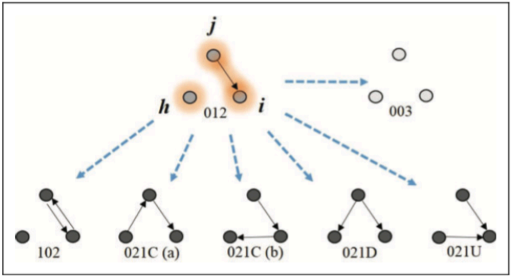
Exponential random graph model (ERGM)

- "Tie-oriented"
- Probability of tie
- Conditional on network structure

Stochastic actor-oriented model (SAOM)

- "Actor-oriented"
- Probability of node forming or maintaining a tie
- Conditional on network structure from a node's perspective

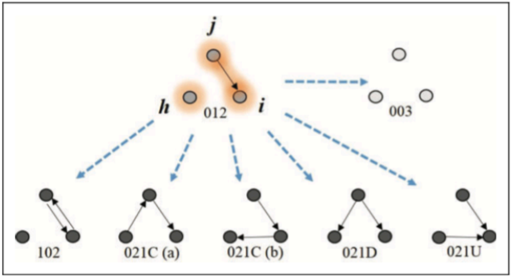
Logic of SAOM (SIENA): tie- versus actor-oriented



Consider change from `012` to `021C(a)` and `021C(b)` with model including density and reciprocity parameter

Block et al. 2016

Logic of SAOM (SIENA): tie- versus actor-oriented



ERGM:

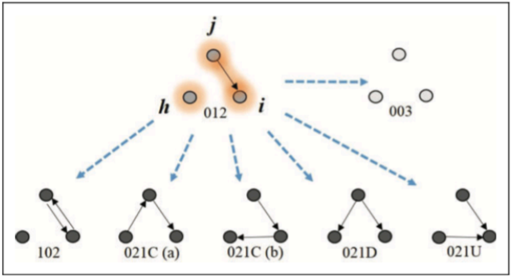
- First fraction: probability to consider tie
- No reciprocity in `021C(a)` and `021C(b)`
- `021C(a)` and `021C(b)` equally likely

$$p_{\mathrm{ERGM}}(012 \rightarrow 021C(a)) = p_{\mathrm{ERGM}}(012 \rightarrow 021C(b))$$

$$= \frac{1}{6} \times \frac{\exp(1 \times \theta_{\text{density}} + 0 \times \theta_{\text{reciprocity}})}{1 + \exp(1 \times \theta_{\text{density}} + 0 \times \theta_{\text{reciprocity}})}$$

$$= \frac{1}{6} \times \frac{\exp(\theta_{\text{density}})}{1 + \exp(\theta_{\text{density}})}$$

Logic of SAOM (SIENA): tie- versus actor-oriented



SAOM:

- First fraction: probability to consider actor
- Reciprocity possible in `021C(b)` but not `021C(a)` (i forms tie)
- If $\beta_{recip} > 0$, `021C(a)` more likely than `021C(b)` (102 more attractive to i)

$$p_{\mathrm{SAOM}}(012 \rightarrow 021C(a)) = \frac{1}{3} \times \frac{\exp(\beta_{\mathrm{den}})}{\exp(0) + \exp(\beta_{\mathrm{den}}) + \exp(\beta_{\mathrm{den}})}$$

and

$$p_{\mathrm{SAOM}}(012 \rightarrow 021C(b)) = \frac{1}{3} \times \frac{\exp(\beta_{\mathrm{den}})}{\exp(0) + \exp(\beta_{\mathrm{den}}) + \exp(\beta_{\mathrm{den}} + \beta_{\mathrm{rec}})}$$

Application: Network homophily

Misery Does Not Love Company: Network Selection Mechanisms and Depression Homophily

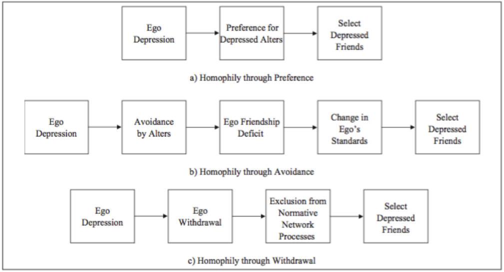
American Sociological Review
76(5) 764–785

DOI: 10.1177/0003122411420813
http://asr.sagepub.com
SAGE

David R. Schaefer,[a] Olga Kornienko,[a] and Andrew M. Fox[a]

What explains depression homophily?

- Test competing mechanisms using SIENA
- Evidence for withdrawal mechanism



Extension: Network interdependence

Development and Psychopathology **26** (2014), 645–659
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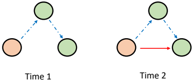
doi:10.1017/S0954579414000297

Victims, bullies, and their defenders: A longitudinal study of the coevolution of positive and negative networks

GIJS HUITSING,[a] TOM A. B. SNIJDERS,[a,b] MARIJTJE A. J. VAN DUIJN,[a] AND RENÉ VEENSTRA[a]
[a]*University of Groningen; and* [b]*University of Oxford*

What is the interplay of bullying and defending victims?

- Dependence across positive and negative networks

Effect name	Formula	Depiction ($t_1 \rightarrow t_2$)
closure	$s_i(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{hj}$	

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What is the interplay of bullying and defending victims?

- Dependence across positive and negative networks
  1. Victims with the same bullies defended each other over time
  2. Defenders become victimized by the bullies of the victims they defend
  3. Defenders of bullies initiated harassment of those bullies' victims

Extension: Selection or influence

Social selection and peer influence in an online social network

Kevin Lewis[a,b,1], Marco Gonzalez[a,c], and Jason Kaufman[b]

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Do people befriend others who are similar to them, or do they become more similar to their friends over time?

- Analyze Facebook data from students in one college
- Use selection-influence SAOM extension by Steglich, Snijders, and Pearson 2010

Network in wave 2 Behavior in wave 1

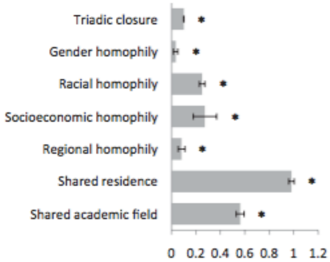
Network objective function effects $\theta. = \beta_h^{net} \quad S. = \sum_m \sum_i s_h^{net}(i, X(t_{m+1}), z(t_m))$

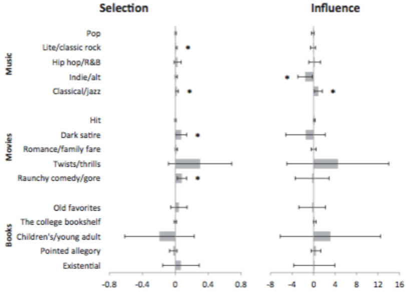
Behavior objective function effects $\theta. = \beta_h^{beh} \quad S. = \sum_m \sum_i s_h^{beh}(i, x(t_m), Z(t_{m+1}))$

Network in wave 1 Behavior in wave 2

Extension: Selection or influence

Do people befriend others who are similar to them, or do they become more similar to their friends over time?





- Network structure and actor attributes influence tie formation
- Shared tastes in music and films, but not books, $\rightarrow$ friendship ties
- Only jazz and classical music tastes spread through friendships

Change we can believe in: Comparing longitudinal network models on consistency, interpretability and predictive power

Per Block, Johan Koskinen, James Hollway, Christian Steglich, Christoph Stadtfeld

Social Networks, 2017

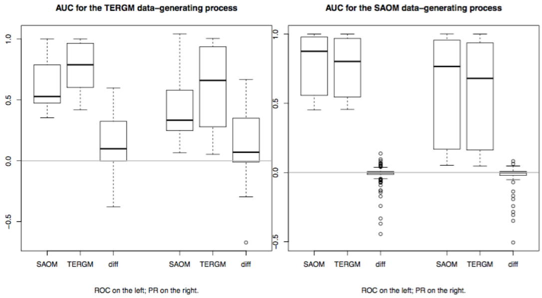
Background

- Cranmer, Leifeld, and Desmarais: ERGMs (and TERGMs) are often better options

  1. *More flexible*: No assumptions about continuity of time, sequentiality, actor choice during data-generating process
  2. *Time dependency is simple*: Condition on previous model realization(s) (*e.g.* dyadic covariate)
     - *More transparent*: Estimate reflects degree of dependence, rather than it being included in updating process
  3. *Model comparison*: TERGM out-predicts SAOM in regards to location of edges in network
     - (T)ERGMs more general
     - SAOM performance depends on DGP meeting model assumptions with precision

Background

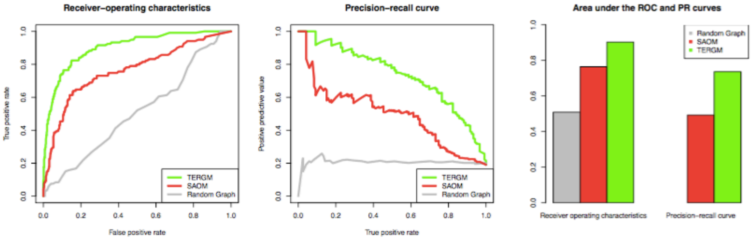
- Leifeld and Cranmer 2016: Predict final observed network wave with model built on temporally previous waves
  - Comparison based on accuracy of predicting edges between each i and j node
  - Compute AUC for receiver operating characteristics (ROC) and precision-recall (PC) curves
- Simulated data using maximally-compatible DGPs:



Background

- Leifeld and Cranmer 2016: Predict final observed network wave with model built on temporally previous waves
  - Comparison based on accuracy of predicting edges between each *i* and *j* node
  - Compute AUC for receiver operating characteristics (ROC) and precision-recall (PC) curves

- Real-world data on school classrooms Snijders et al. 2010:



Change we can believe in: Argument

1. How do discrete-time and continuous time models treat time?
   a. Modeling time
   b. Interpreting time
2. Model performance and prediction

1a. Treatment of time

Discrete-time models (TERGM):

- Answer questions about structure
- “What regularities does the network at time t_m exhibit, taking into account information about t_{m-1}?”

Continuous-time models (SAOM):

- Answer questions about change
- “According to which regularities does the network evolve from t_{m-1} to t_m?”

1a. Treatment of time

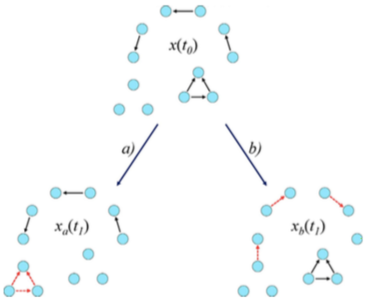


Fig. 1: Red = changing ties; black = stable ties

- **TERGM**: Both subsequent network states have equal probability ; structural features are the same between $x_a(t_1)$ and $x_b(t_1)$ (*e.g.*, parameter for transitive triad will be identical)
- **SAOM**: Modeling mini-steps will show very different processes leading to $x_a(t_1)$ and $x_b(t_1)$ (*e.g.*, parameter for TT will be larger for $x_a(t_1)$)

1a. Treatment of time

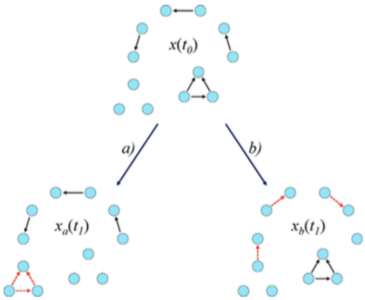


Fig. 1: Red = changing ties; black = stable ties

- **TERGM**: Time is decoupled from dependence — one parameter models time (*i.e.*, auto-regressive term) and another models specific structures in current network
- **SAOM**: Dependence unfolds over time — modeling of mini-steps

1b. Interpretation of time

SAOM:

- *Interpretation at level of mini-step*: Which new types of ties (*e.g.*, reciprocated ties) are more/less likely to exist at next step
- Description of social mechanisms
- Under assumption that real-world process approximated by sequentiality

1b. Interpretation of time

TERGM:

- Auto-regressive term includes same network as present network
- *Interpretation at network level*: Probability to observe more/less of pattern of ties than expected by chance, conditional on past *and* tendency of same pattern in the past

1b. Interpretation of time

TERGM:

- Auto-regressive term includes same network as present network
- *Interpretation at tie level*: Not possible because equilibrium assumption is violated
  - ERGM: Parameters indicate tendency for dyads to form types of tie configurations (*e.g.*, reciprocity)
  - Necessary assumption that network is in equilibrium (initial state irrelevant)
  - TERGM: Inclusion of past information contradicts assumption of time independence
  - No micro-level interpretation hinders insights into change and identification of social mechanisms

1b. Importance of time

How does time elapsed affect parameters?

- **Continuous time**: Effect parameters independent of time
  - More time → more simulated mini-steps (upon which parameters are based)

- **Discrete time**: Parameters not independent of time passage
  1. Assume $x(t_0)$ network with tendency towards reciprocity
  2. If little time until $x(t_1)$, reciprocity parameter will be small
     - Change explained by stability parameter (*i.e.*, information from $x(t_0)$)
  3. As more time passes (and change happens), reciprocity parameter will increase
     - Less change explained by information from $x(t_0)$

1b. Illustration of time dependence

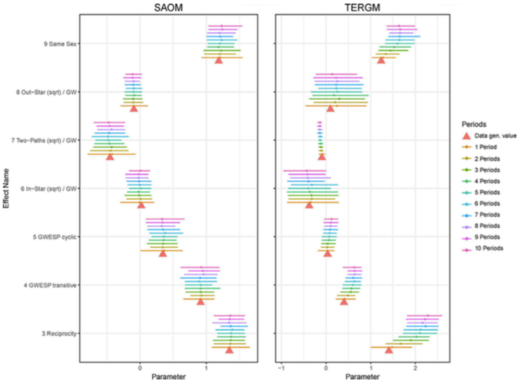


Fig. 2. Estimated Parameters and 90% range of re-estimation of the Glasgow data.

1. Estimate models based on empirical data
2. Parameters used to simulate 100 replicates of 10 waves
3. Pair of simulated waves used to re-estimate models
4. Indication of parameter stability across varying time spans

- **SAOM**: Parameter estimates consistent across elapsed time
- **TERGM**: Parameter estimates sensitive to time between observations; Recording network at “correct” time-points become important

Performance and prediction

Cross-validation is powerful way to assess model, but…

…for networks, evaluate based on dependence or specific ties?

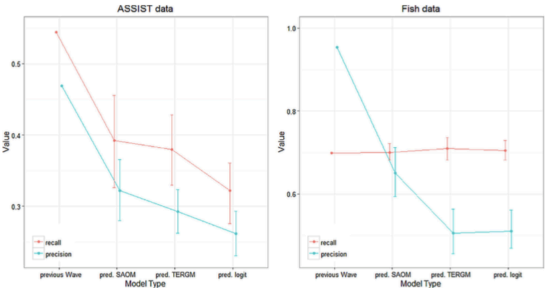
1. Already common to evaluate based on dependence
   - Simulate data from fitted model, compare to "auxiliary statistics" of observed data
   - Also possible to use predictive distributions of out-of-sample data Koskinen and Snijders 2007
   - But not appropriate for comparing ERGM and SAOM: Modeling of dependence differs (recall mathematical meaning of "actor-oriented", and see Block et al. 2016)
2. Can prediction of specific ties be useful criterion to compare performance? Leifeld and Cranmer 2016

Performance and prediction

Claim: *Based on theoretical reasoning, predicting ties is not a useful evaluation criterion.*

1. Models will be bad at predicting the rare event of specific ties
2. TERGM is not an improvement over a logistical regression (without structural effects) for predicting tie location, *unless* additional statistics are included to constrain location of ties
   Fig. 3:
   - TERGM concerned with network-level statistics (*e.g.*, 5 ties, 1 reciprocated tie, 2 pre-existing ties). . .
   - . . . But, specific location of these, *e.g.*, 5 ties will be weighted by TERGM with stability term (*i.e.*, previous wave)
   - So, sometimes TERGM will be good at tie-location prediction, sometimes not
   - Including structural information for predicted wave improves prediction; requires knowing the future

Performance and prediction



1. Fit TERGM and SAOM model to first two waves
2. Logit w/o structural effects
3. Trival "persistence" model
4. Simulate 1000 wave 3 networks; compare to observed data
5. *Precision*: correctly predicted ties / total predicted ties
6. *Recall*: correctly predicted ties / total observed ties

- TERGM or SAOM not good at predicting tie location
- Tie location should not be a criterion of comparison (contra Leifeld and Cranmer 2016)

Conclusion

- Discrete- versus continuous-time longitudinal network models
- Differences in interpretability and treatment of time
- SAOM / SIENA best for micro-level social mechanisms
- Key issues:
  - Conceptualizing time
  - Using prediction