Soc500: Applied Social Statistics
Week 1: Introduction and Probability

Brandon Stewart[1]

Princeton

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[1] These slides are heavily influenced by Matt Blackwell and Adam Glynn with contributions from Justin Grimmer and Matt Salganik. Illustrations by Shay O'Brien.

Where We've Been and Where We're Going...

- Last Week
  - methods camp
  - pre-grad school life
- This Week
  - Wednesday
    - welcome
    - basics of probability
- Next Week
  - random variables
  - joint distributions
- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causal inference

Questions?

Welcome and Introductions

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  - Shay O'Brien (Soc500)
  - Alex Kindel (Soc400)
  - Ziyao Tian (Soc400)

Overview

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- Syllabus is a useful resource including philosophy of the class.

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  - write clean, reusable, and reliable R code.
  - feel empowered working with data

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Why RMarkdown?

What you've done before

Image Credit: Baumer et al (2014)

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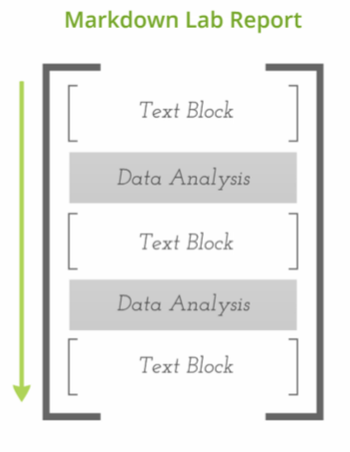

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- We all come from very different backgrounds. Please have patience with yourself and with others.

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Your Job: work hard and get help when you need it!

Daily Feedback

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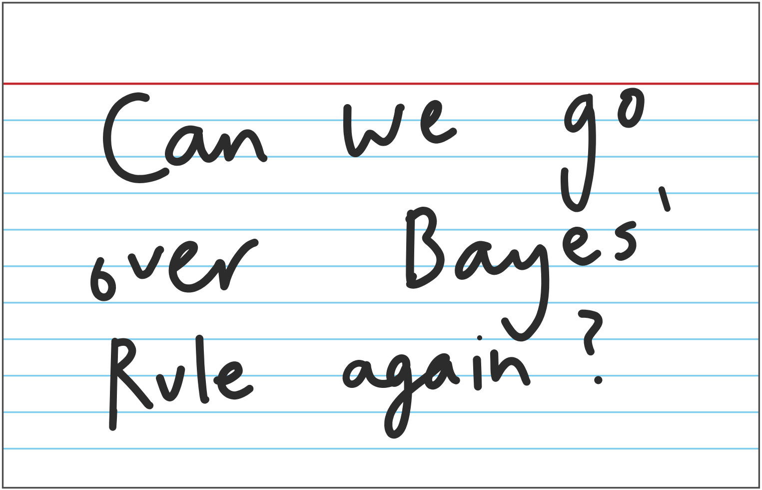

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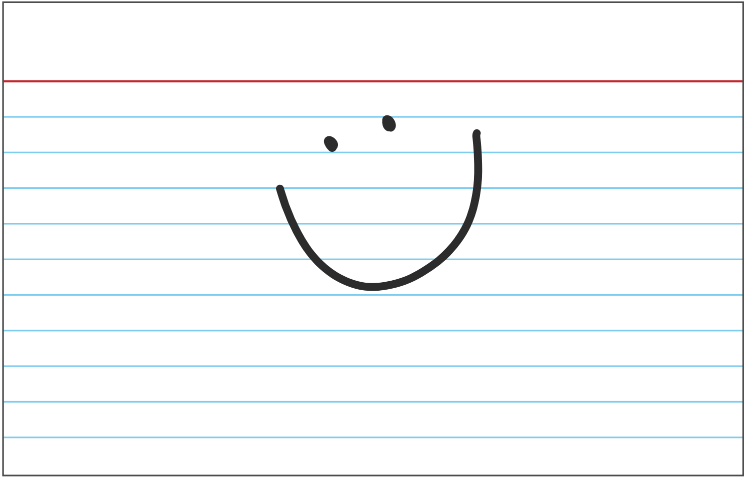

How to Get Help

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2. Daily Feedback
3. Readings and Slides
4. Piazza
5. Preceptor Office Hours
6. Instructor Office Hours

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Read the syllabus for more details.

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- It's challenging but very doable and rewarding if you put the time in. There are plenty of resources to take advantage of for help.
- This course is very challenging but greatly contributed to my understanding of social statistics. If you're truly invested in the subject and willing to put in the work (more than you expect possibly), it will be one of the best courses you've taken.

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Probability $\rightarrow$ Inference $\rightarrow$ Regression $\rightarrow$ Causal Inference

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This Class

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Let's get started

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- In practice, an essential part of research, policy making, political campaigns, selling people things. . .

Why study probability?

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It enables inference

In Picture Form

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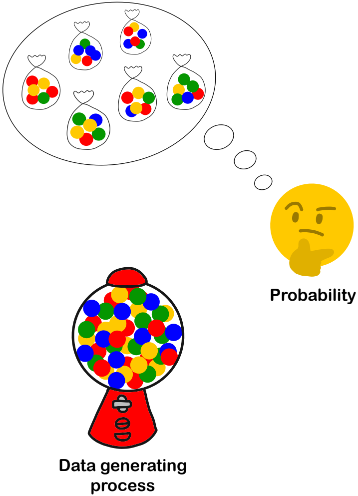

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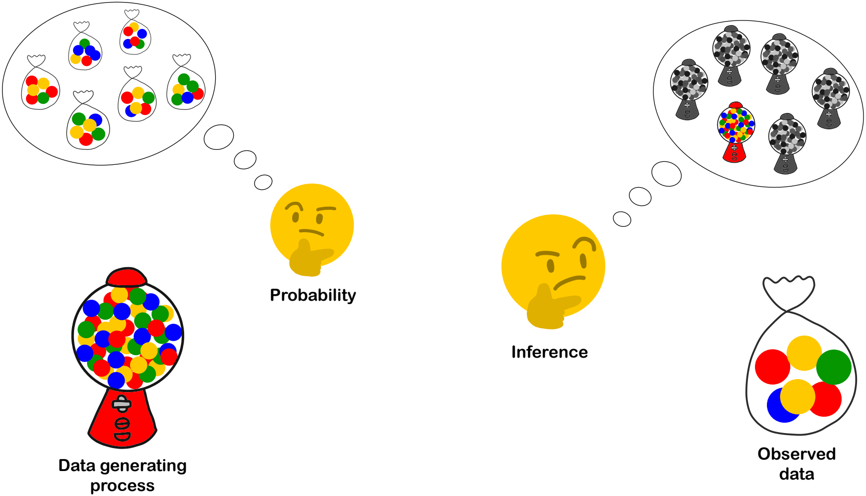

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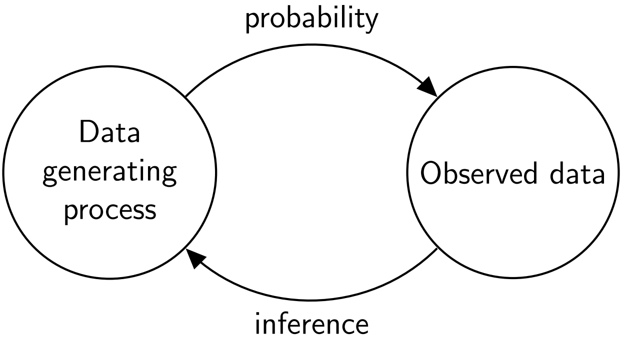

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  - it tells us what the world would look like under a certain assumption
- We will review probability today, but feel free to ask questions as needed throughout the semester.

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- The Story Setup

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  (lady discerning about tea)

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- The Hypothetical

Tea-Tasting Distribution

Success count	Permutations of selection	Number of permutations
0	oooo	1 × 1 = 1
1	ooox, ooxo, oxoo, xooo	4 × 4 = 16
2	ooxx, oxox, oxxo, xoxo, xxoo, xoox	6 × 6 = 36
3	oxxx, xoxx, xxox, xxxo	4 × 4 = 16
4	xxxx	1 × 1 = 1
Total		70

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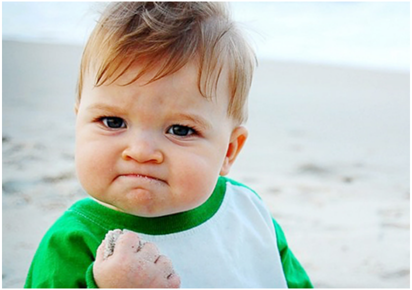
- The Result



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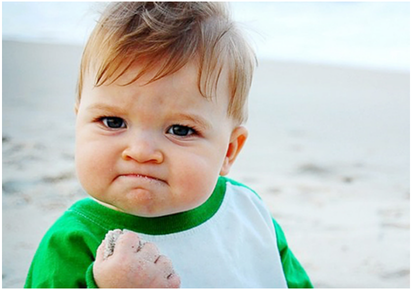

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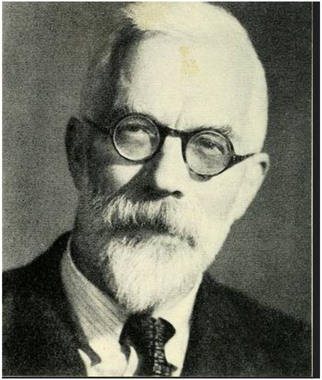
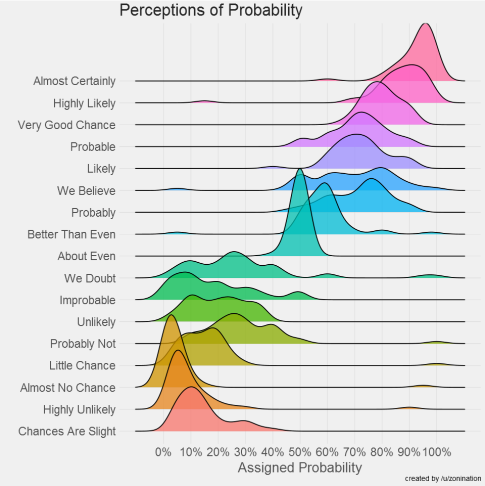


This became the Fisher Exact Test.

From 'Probably' to Probability



Can we make this more precise?

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- Thus: we need to know how probability gives rise to data

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All the rules of probability can be derived from these axioms.
(we will return to these in a minute)

Sample Spaces

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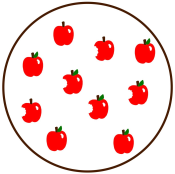

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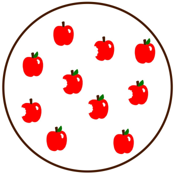

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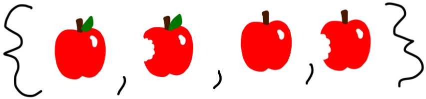
$\Omega = \mathbf{S} =$ 

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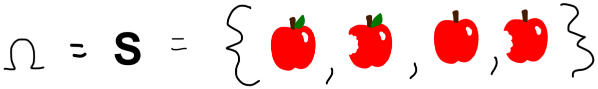

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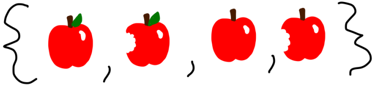
Ω = **S** = 

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One way to define an event is to describe the common property that all of the outcomes share. We write this as

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If $A = \{\omega | \omega \text{ has a leaf}\}$:

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Complement

A complement of event A is a set: A^c, is collection of all of the outcomes not in A. That is, it is "everything else" in the sample space.

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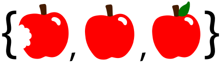

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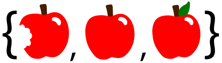

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Important complement: $\Omega^c = \emptyset$, where $\emptyset$ is the empty set—it's just the event that nothing happens.

Unions and intersections (Operations on events)

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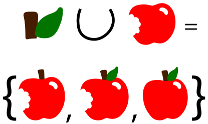
The union of two events, A and B is the event that A or B occurs:



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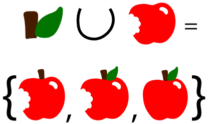

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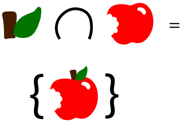

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We say that two events A and B are disjoint or mutually exclusive if they don't share any elements or that $A \cap B = \emptyset$.

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Sample spaces can have infinite events $A_1, A_2, \ldots$.

Probability Function

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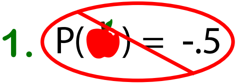

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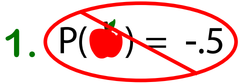

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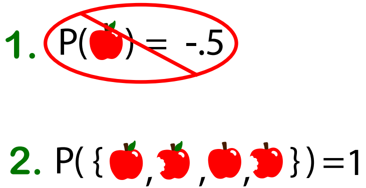

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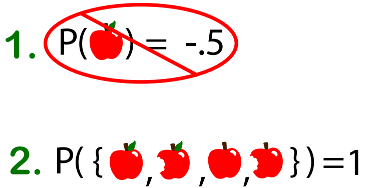

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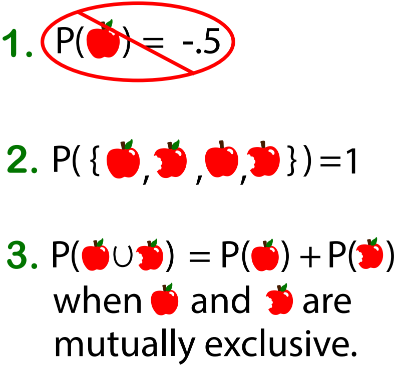

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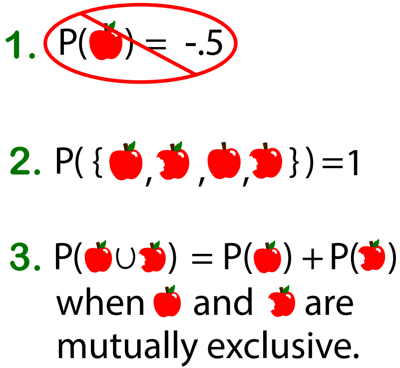

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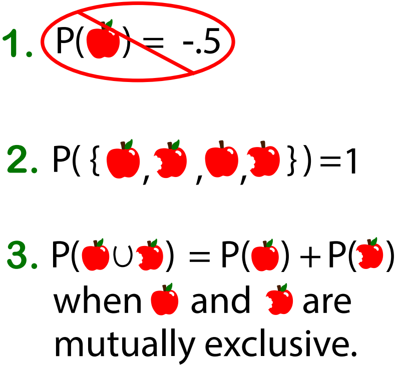

All the rules of probability can be derived from these axioms. (See Blitzstein & Hwang, Def 1.6.1.)

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Three Big Ideas

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Independence

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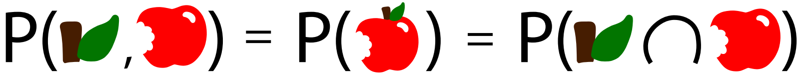
Suppose we are now in a situation where we would like to express the probability that an event A and an event B occur. This quantity is written as $P(A \cap B)$, $P(B \cap A)$, $P(A, B)$, or $P(B, A)$ and is the joint probability of A and B.



P() = ?

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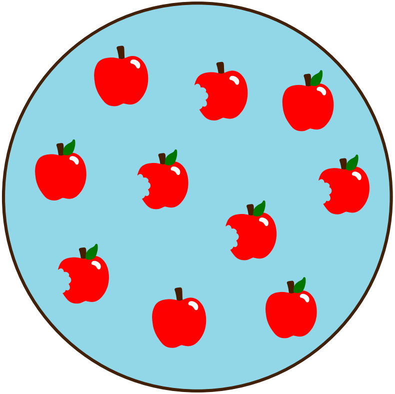

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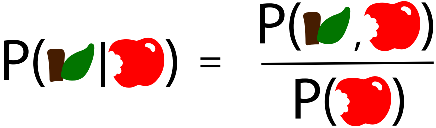
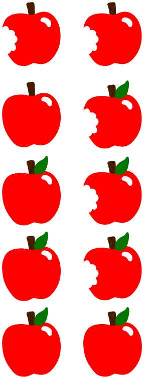
Conditional Probability: A Visual Example





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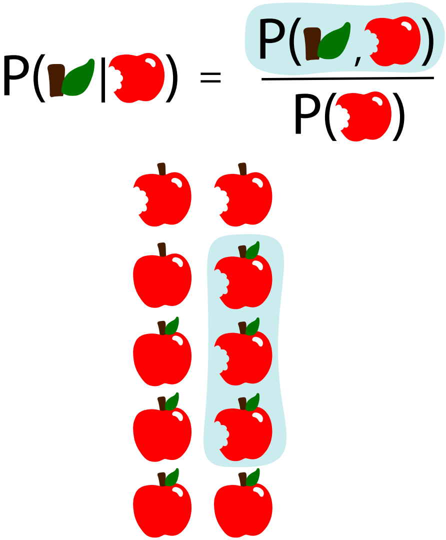

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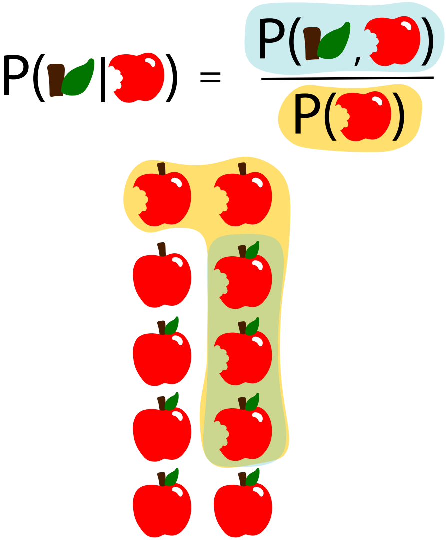

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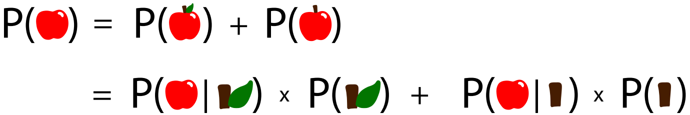
Law of Total Probability (LTP)

With 2 Events:

$$\begin{aligned} P(B) &= P(B, A) + P(B, A^c) \\ &= P(B|A) \times P(A) + P(B|A^c) \times P(A^c) \end{aligned}$$



Recall, if we randomly draw two cards from a standard 52 card deck and define the events $A = \{\text{King on Draw 1}\}$ and $B = \{\text{King on Draw 2}\}$, then

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Question: $P(B) =?$

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$$
\begin{aligned}
P(B) &= 3/51 \times 1/13 + 4/51 \times 12/13 \\
&= \frac{3 + 48}{51 \times 13} = \frac{1}{13} = \frac{4}{52}
\end{aligned}
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- $\Pr(\text{mobilized}) = 0.6$ and so $\Pr(\text{ not mobilized}) = 0.4$

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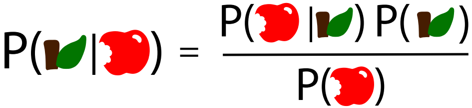
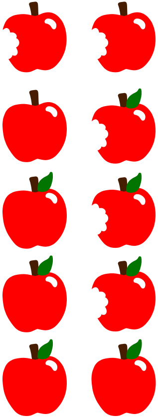
- Proof: combine the multiplication rule $\Pr(B|A)\Pr(A) = P(A \cap B)$, and the definition of conditional probability

Bayes' Rule Mechanics





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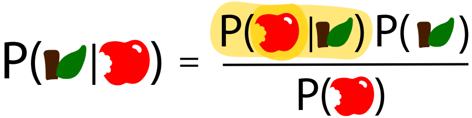

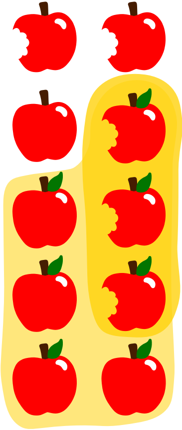

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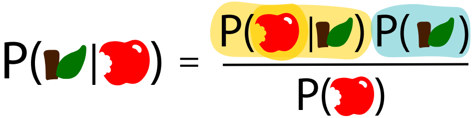

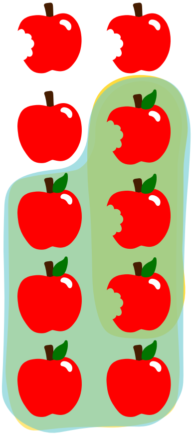

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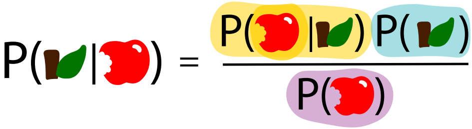
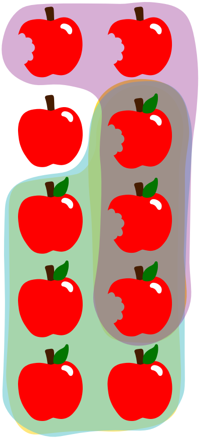
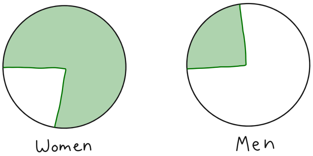




Bayes' Rule Example

U.S. Billionaires, 2014



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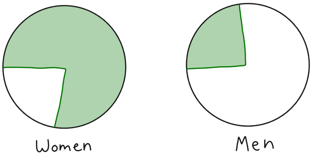

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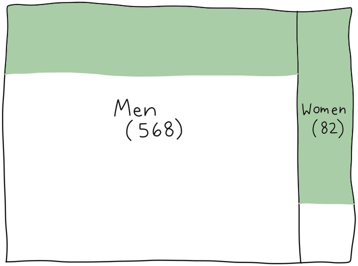
$$P(W \mid I) = \frac{P(I \mid W)\,P(W)}{P(I)}$$

$$= \frac{.765\left(\frac{82}{568+82}\right)}{\frac{.765(82) + .245(568)}{568+82}}$$

$$= .31$$



* Data source = Billionaires characteristics database

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&\approx 0.9
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Independence is a massively important concept in statistics.

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- A word from your preceptors

References

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- Salsburg, David. *The Lady Tasting Tea: How Statistics Revolutionized Science in the Twentieth Century* (2002).