Week 10: Causality with Measured Confounding

Brandon Stewart[1]

Princeton

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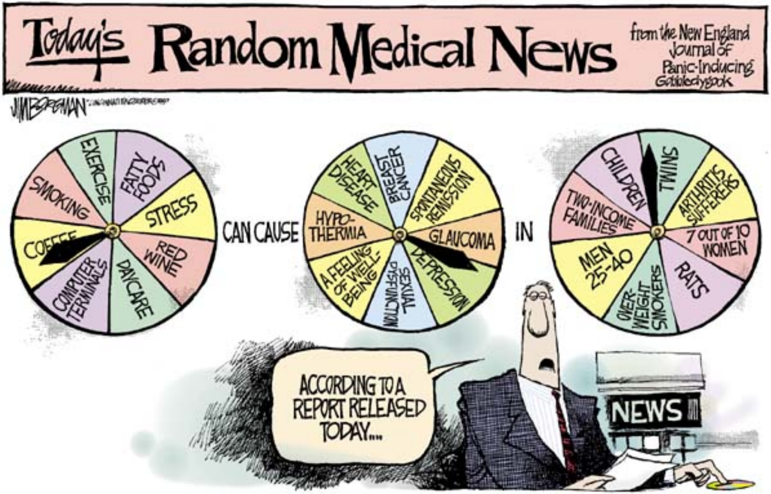
[1] These slides are heavily influenced by Matt Blackwell, Jens Hainmueller, Erin Hartman, Kosuke Imai and Gary King.

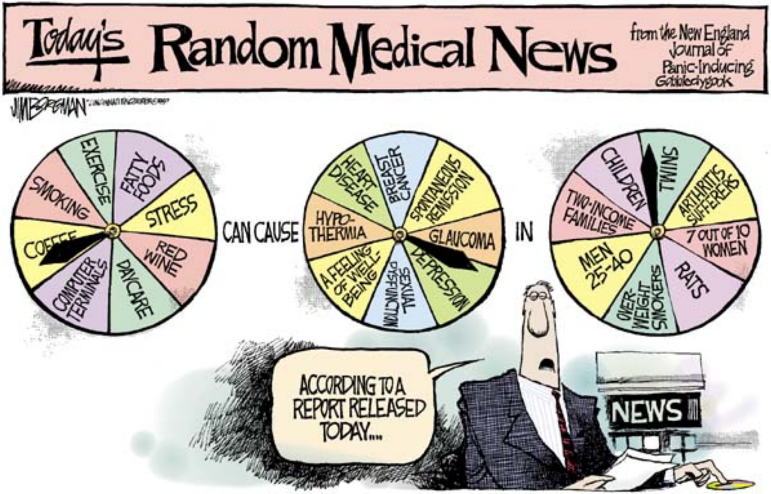
Where We've Been and Where We're Going...

- Last Week
  - intro to causal inference
- This Week
  - Monday:
    - experimental Ideal
    - identification with measured confounding
  - Wednesday:
    - regression estimation
- Next Week
  - identification with unmeasured confounding
  - instrumental variables
- Long Run
  - probability → inference → regression → causal inference

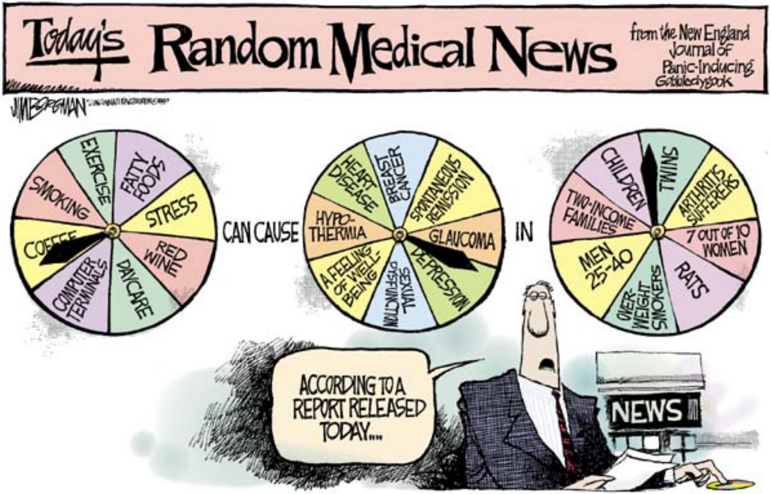
Questions?



Lancet 2001: negative correlation between coronary heart disease mortality and level of vitamin C in bloodstream (controlling for age, gender, blood pressure, diabetes, and smoking)



Lancet 2002: no effect of vitamin C on mortality in controlled placebo trial (controlling for nothing)



Lancet 2003: comparing among individuals with the same age, gender, blood pressure, diabetes, and smoking, those with higher vitamin C levels have lower levels of obesity, lower levels of alcohol consumption, are less likely to grow up in working class, etc.

Why So Much Variation?

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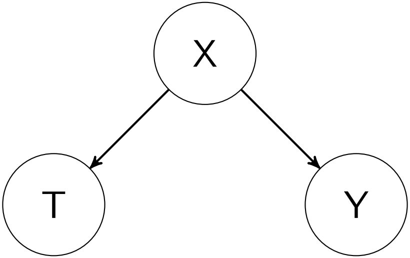
Confounders



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- Cannot always randomize so we do observational studies, where we adjust for the observed covariates and hope that unobservables are balanced
- Better than hoping: design observational study to approximate an experiment
  - "The planner of an observational study should always ask himself: How would the study be conducted if it were possible to do it by controlled experimentation" (Cochran 1965)

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   - Treatment assignment does not depend on any potential outcomes.
   - Sometimes written as $D_i \perp\!\!\!\perp (\mathbf{Y}(1), \mathbf{Y}(0))$

Why do Experiments Help?

Remember selection bias?

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When all goes well, an experiment eliminates selection bias.

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  - Try to estimate the randomization procedure.
  - Analyze this as an experiment with this estimated procedure.
- Tries to minimize “snooping” by picking the best modeling strategy before seeing the outcome.

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- Never used our knowledge of the randomization for this quantity.

Stratification Example: Smoking and Mortality (Cochran, 1968)

TABLE 1

DEATH RATES PER 1,000 PERSON-YEARS

Smoking group	Canada	U.K.	U.S.
Non-smokers	20.2	11.3	13.5
Cigarettes	20.5	14.1	13.5
Cigars/pipes	35.5	20.7	17.4

Stratification Example: Smoking and Mortality (Cochran, 1968)

TABLE 2

MEAN AGES, YEARS

Smoking group	Canada	U.K.	U.S.
Non-smokers	54.9	49.1	57.0
Cigarettes	50.5	49.8	53.2
Cigars/pipes	65.9	55.7	59.7

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- calculate death rates within age subgroups
- average within age subgroup death rates using fixed weights (e.g. number of cigarette smokers)

Stratification: Example

	Death Rates Pipe Smokers	# Pipe-Smokers	# Non-Smokers
Age 20 - 50	15	11	29
Age 50 - 70	35	13	9
Age + 70	50	16	2
Total		40	40

What is the average death rate for Pipe Smokers?

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$15 \cdot (29/40) + 35 \cdot (9/40) + 50 \cdot (2/40) = 21.2$

Smoking and Mortality (Cochran, 1968)

TABLE 3

ADJUSTED DEATH RATES USING 3 AGE GROUPS

Smoking group	Canada	U.K.	U.S.
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Cigarettes	28.3	12.8	17.7
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One option is to discretize as we discussed with age, we will discuss more later this week!

Identification Under Selection on Observables

Identification Assumption

1. $(Y_1, Y_0) \perp\!\!\!\perp D | X$ *(selection on observables)*
2. $0 < \Pr(D = 1|X) < 1$ *with probability one (common support)*

Identification Result

Given selection on observables we have

$$\begin{aligned} \mathbb{E}[Y_1 - Y_0|X] &= \mathbb{E}[Y_1 - Y_0|X, D = 1] \\ &= \mathbb{E}[Y|X, D = 1] - \mathbb{E}[Y|X, D = 0] \end{aligned}$$

Therefore, under the common support condition:

$$\begin{aligned} \tau_{ATE} &= \mathbb{E}[Y_1 - Y_0] = \int \mathbb{E}[Y_1 - Y_0|X]\, dP(X) \\ &= \int \big(\mathbb{E}[Y|X, D = 1] - \mathbb{E}[Y|X, D = 0]\big)\, dP(X) \end{aligned}$$

Identification Under Selection on Observables

Identification Assumption

1. $(Y_1, Y_0) \perp\!\!\!\perp D|X$ *(selection on observables)*
2. $0 < \Pr(D = 1|X) < 1$ *with probability one (common support)*

Identification Result

Similarly,

$$\begin{aligned} \tau_{ATT} &= \mathbb{E}[Y_1 - Y_0|D = 1] \\ &= \int \left(\mathbb{E}[Y|X, D = 1] - \mathbb{E}[Y|X, D = 0]\right) dP(X|D = 1) \end{aligned}$$

To identify τ_{ATT} the selection on observables and common support conditions can be relaxed to:

- $Y_0 \perp\!\!\!\perp D|X$ *(SOO for Controls)*
- $\Pr(D = 1|X) < 1$ *(Weak Overlap)*

Identification Under Selection on Observables

unit	Potential Outcome under Treatment	Potential Outcome under Control		
i	Y_{1i}	Y_{0i}	D_i	X_i
1	$\mathbb{E}[Y_1 \mid X=0, D=1]$	$\mathbb{E}[Y_0 \mid X=0, D=1]$	1	0
2			1	0
3	$\mathbb{E}[Y_1 \mid X=0, D=0]$	$\mathbb{E}[Y_0 \mid X=0, D=0]$	0	0
4			0	0
5	$\mathbb{E}[Y_1 \mid X=1, D=1]$	$\mathbb{E}[Y_0 \mid X=1, D=1]$	1	1
6			1	1
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$(Y_1, Y_0) \perp\!\!\!\perp D \mid X$ implies that we conditioned on all confounders. The treatment is randomly assigned within each stratum of X:

$$\mathbb{E}[Y_0 \mid X=0, D=1] = \mathbb{E}[Y_0 \mid X=0, D=0] \text{ and}$$
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$(Y_1, Y_0) \perp\!\!\!\perp D \mid X$ also implies

$$\begin{aligned} \mathbb{E}[Y_1 \mid X=0, D=1] &= \mathbb{E}[Y_1 \mid X=0, D=0] \text{ and} \\ \mathbb{E}[Y_1 \mid X=1, D=1] &= \mathbb{E}[Y_1 \mid X=1, D=0] \end{aligned}$$

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- No unmeasured confounding assumes that we've measured all sources of confounding.

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- Another way: use DAGs and look at back-door paths.

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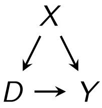
- Backdoor paths between D and Y $\rightsquigarrow$ common causes of D and Y:



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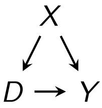
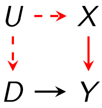


- Here there is a backdoor path $D \leftarrow X \rightarrow Y$, where X is a common cause for the treatment and the outcome.

Other types of confounding



- D is enrolling in a job training program.

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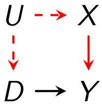

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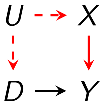

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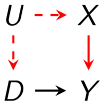

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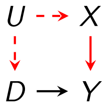

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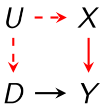
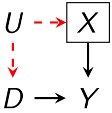


- D is exercise.
- Y is having a disease.
- U is lifestyle.
- X is smoking
- Big assumption here: no arrow from U to Y.

What's the problem with backdoor paths?



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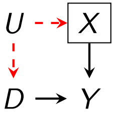

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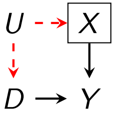

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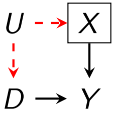

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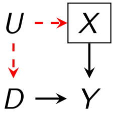

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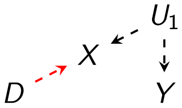
- In the DAG here, if we condition on X, then the backdoor path is blocked.

Not all backdoor paths



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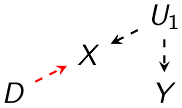

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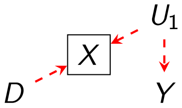

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Don't condition on post-treatment variables

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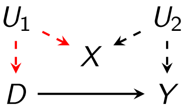
Every time you do, a puppy cries.

M-bias



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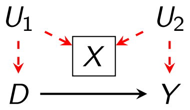

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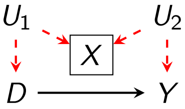

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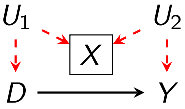

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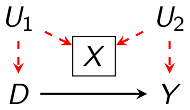

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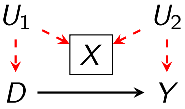

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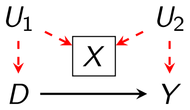

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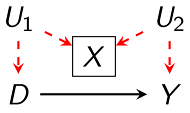

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  - See the Elwert and Winship piece for more!

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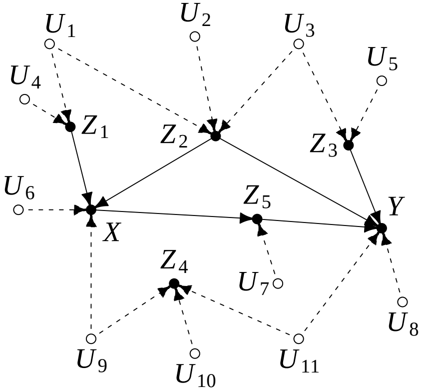
  - what variables to condition on to eliminate the confounding.

Example: Sufficient Conditioning Sets



Remove arrows out of X.

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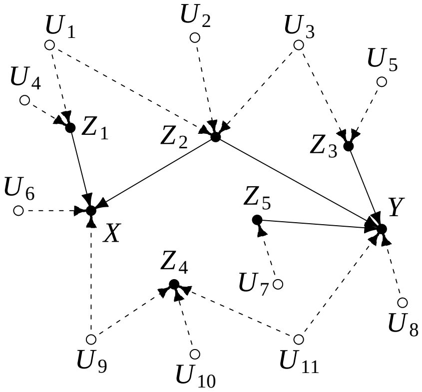

Recall that paths are blocked by "unconditioned colliders" or conditioned non-colliders

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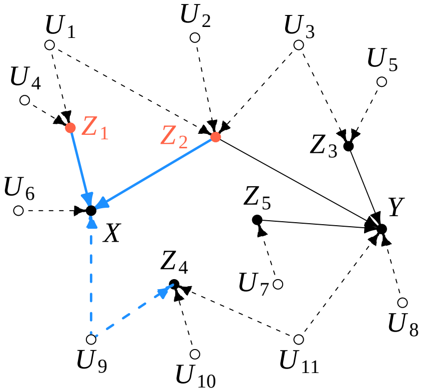

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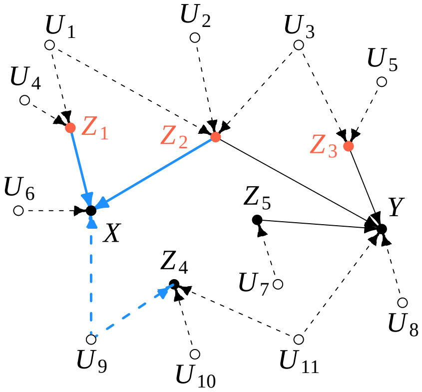

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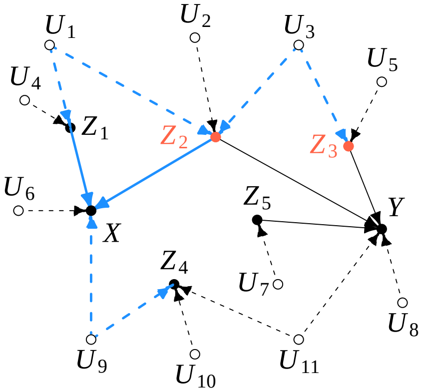

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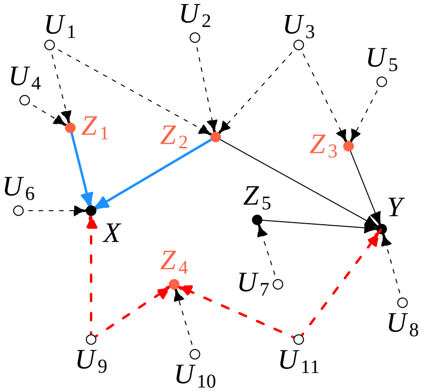

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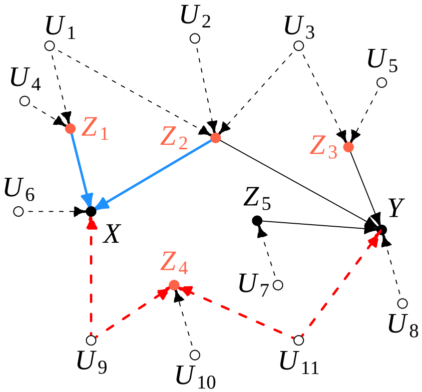

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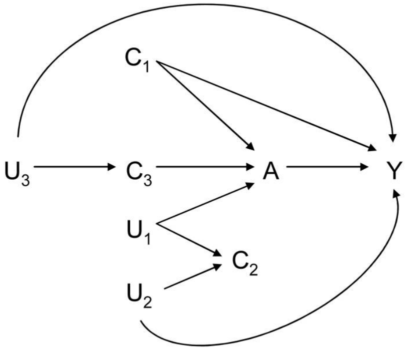

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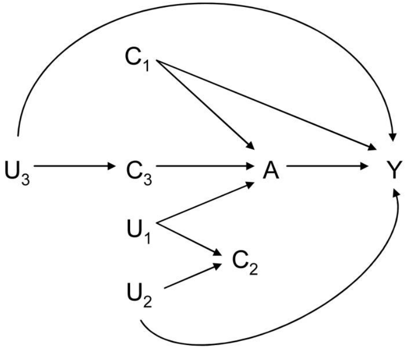

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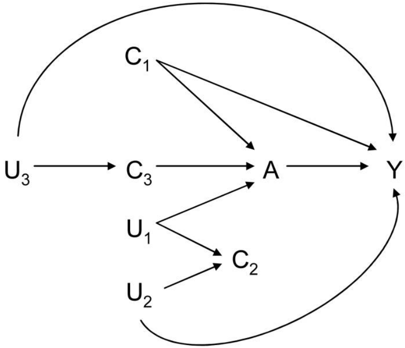

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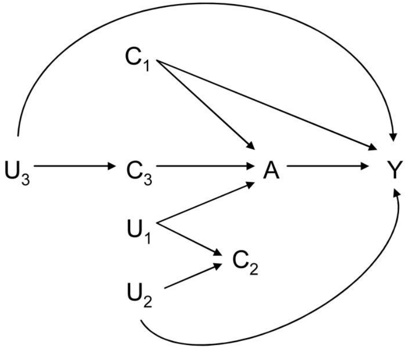

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Assessing no unmeasured confounders

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THE FOX NEWS EFFECT: INTERACTIONS AND PLACEBO SPECIFICATIONS

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Dep. var.	(1)	(2)	(3)	(4)	(5)
Availability of Fox News via cable in 2000	0.0109 (0.0042)***	0.0105 (0.0039)***	0.0036 (0.0016)**	−0.0024 (0.0031)	0.0026 (0.0026)
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- Unconfoundedness could still be violated even if you pass this test!

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  - All discussed in the next couple of weeks!

Where We've Been and Where We're Going...

- Last Week
  - intro to causal inference
- This Week
  - Monday:
    - experimental Ideal
    - identification with measured confounding
  - Wednesday:
    - regression estimation
- Next Week
  - identification with unmeasured confounding
  - instrumental variables
- Long Run
  - probability → inference → regression → causal inference

Questions?

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- Identification depends on assumptions not statistical models.

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- Today we will talk about regression because that's the subject of the class.
- A big topic I'm skipping over as outside the scope of class is the propensity score (conditional expectation of the treatment given the covariates).

Regression

David Freedman:

> *I sometimes have a nightmare about Kepler. Suppose a few of us were transported back in time to the year 1600, and were invited by the Emperor Rudolph II to set up an Imperial Department of Statistics in the court at Prague. Despairing of those circular orbits, Kepler enrolls in our department. We teach him the general linear model, least squares, dummy variables, everything. He goes back to work, fits the best circular orbit for Mars by least squares, puts in a dummy variable for the exceptional observation - and publishes. And that's the end, right there in Prague at the beginning of the 17th century.*

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- When is regression causal? When the CEF is causal.
- This means that the question of whether regression has a causal interpretation is a question about identification

Identification under Selection on Observables: Regression

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   - τ will provide unbiased and consistent estimates of ATE.
2. Constant treatment effects and unknown functional form
   - τ will provide well-defined linear approximation to the average causal response function $\mathbb{E}[Y|D=1,X] - \mathbb{E}[Y|D=0,X]$. Approximation may be very poor if $\mathbb{E}[Y|D,X]$ is misspecified and then τ may be biased for the ATE.

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3. Heterogeneous treatment effects (τ differs for different values of X)
   - If outcomes are linear in X, τ is unbiased and consistent estimator for conditional-variance-weighted average of the underlying causal effects. This averagecan be different from the ATE.

Identification under Selection on Observables: Regression

Identification Assumption

1. *Constant treatment effect:* $\tau = Y_{1i} - Y_{0i}$ *for all* i
2. *Control outcome is linear in* X: $Y_{0i} = \beta_0 + X_i'\beta + \epsilon_i$ *with* $\epsilon_i \perp\!\!\!\perp X_i$ *(no omitted variables and linearly separable confounding)*

Identification Result

Then $\tau_{ATE} = \mathbb{E}[Y_1 - Y_0]$ *is identified by a regression of the observed outcome on the covariates and the treatment indicator*
$Y_i = \beta_0 + \tau D_i + X_i'\beta + \epsilon_i$

Ideal Case: Linear Constant Effects Model

Assume constant linear effects and linearly separable confounding:

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- Thus, a regression where D_i and X_i are entered linearly can recover the ATE.

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- Constant effects and linearly separable confounding aren't very appealing or plausible assumptions
- To understand what happens when they don't hold, we need to understand the properties of regression with minimal assumptions: this is often called an agnostic view of regression.
- The Aronow and Miller book (and lecture 7) provide some context but essentially as long as we have iid sampling, we will asymptotically obtain the best linear approximation to the CEF.

1. The Experimental Ideal
2. Assumption of No Unmeasured Confounding
3. Estimation Under No Unmeasured Confounding
4. Regression Estimators
5. Regression and Causality
6. Regression Under Heterogeneous Effects
7. Fun with Visualization, Replication and the NYT

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- The question, then, is when does knowing the CEF tell us something about causality?
- Angrist and Pishke argues that a regression is causal when the CEF it approximates is causal. Identification is king.
- We will show that under certain conditions, a regression of the outcome on the treatment and the covariates can recover a causal parameter, but perhaps not the one in which we are interested.

Linear constant effects model, binary treatment

Now with the benefit of covering agnostic regression, let's review again the simple case.

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$$\begin{aligned} Y_i &= D_i Y_i(1) + (1 - D_i) Y_i(0) \\ &= Y_i(0) + (Y_i(1) - Y_i(0)) D_i \\ &= \mathbb{E}[Y_i(0)] + \tau D_i + (Y_i(0) - \mathbb{E}[Y_i(0)]) \\ &= \mu^0 + \tau D_i + v_i^0 \end{aligned}$$

- Note that if ignorability holds (as in an experiment) for $Y_i(0)$, then it will also hold for v_i^0, since $\mathbb{E}[Y_i(0)]$ is constant. Thus, this satifies the usual assumptions for regression.

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- Consistency assumption allows us to write this as:

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&= \alpha + \tau D_i + X_i'\gamma
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- Works with continuous or ordinal D_i if effect of these variables is truly linear.

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- Thus, OLS estimates the ATE with no covariates.

Adding covariates

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- What about the regression estimand, τ_R? How does it relate to the ATE/ATT?

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- Let's investigate this under a saturated regression model:

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- Linear in X_i by construction!

Investigating the regression coefficient

- How can we investigate τ_R? Well, we can rely on the regression anatomy:
$$\tau_R = \frac{\text{Cov}(Y_i, D_i - E[D_i|X_i])}{\mathrm{Var}(D_i - E[D_i|X_i])}$$

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$$\tau_R = \frac{\mathbb{E}\left[\tau(X_i)(D_i - \mathbb{E}[D_i|X_i])^2\right]}{\mathbb{E}[(D_i - E[D_i|X_i])^2]} = \frac{\mathbb{E}[\tau(X_i)\sigma_d^2(X_i)]}{\mathbb{E}[\sigma_d^2(X_i)]}$$

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- $\sigma_d^2(x) = \text{Var}[D_i|X_i = x]$ is the conditional variance of treatment assignment.

ATE versus OLS

$$\tau_R = \mathbb{E}[\tau(X_i)W_i] = \sum_x \tau(x)\frac{\sigma_d^2(x)}{\mathbb{E}[\sigma_d^2(X_i)]}\mathbb{P}[X_i = x]$$

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- The ATE weights only by their size.

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- Maximum variance with $\mathbb{P}[D_i = 1|X_i = x] = 1/2$.

OLS weighting example

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- $\rightsquigarrow$ weights for $X_i = 1$ are: $0.09/0.13 = 0.692$, for $X_i = 0$: $0.25/0.13 = 1.92$.

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OLS weighting example

- Binary covariate:

	Group 1	Group 2
	$\mathbb{P}[X_i = 1] = 0.75$	$\mathbb{P}[X_i = 0] = 0.25$
	$\mathbb{P}[D_i = 1\vert X_i = 1] = 0.9$	$\mathbb{P}[D_i = 1\vert X_i = 0] = 0.5$
	$\sigma^2_d(1) = 0.09$	$\sigma^2_d(0) = 0.25$
	$\tau(1) = 1$	$\tau(0) = -1$

- Implies the ATE is $\tau = 0.5$
- Average conditional variance: $\mathbb{E}[\sigma^2_d(X_i)] = 0.13$
- $\rightsquigarrow$ weights for $X_i = 1$ are: $0.09/0.13 = 0.692$, for $X_i = 0$: $0.25/0.13 = 1.92$.

$$\begin{aligned}\tau_R &= \mathbb{E}[\tau(X_i)W_i] \\ &= \tau(1)W(1)\mathbb{P}[X_i = 1] + \tau(0)W(0)\mathbb{P}[X_i = 0] \\ &= 1 \times 0.692 \times 0.75 + -1 \times 1.92 \times 0.25\end{aligned}$$

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- Incorrect linearity assumption in X_i will lead to more bias.

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- How can we use this?

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- Impute the treated potential outcomes with $\widehat{Y}_i(1) = \hat{\mu}_1(X_i)$!

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- Sometimes called an **imputation estimator**.

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- Useful trick: use a model on the entire data and `model.frame()` to get the right design matrix:

```
## heterogeneous effects
y.het <- ifelse(d == 1, y + rnorm(n, 0, 5), y)

mod <- lm(y.het ~ d + X)
mod1 <- lm(y.het ~ X, subset = d == 1)
mod0 <- lm(y.het ~ X, subset = d == 0)
y1.imps <- predict(mod1, model.frame(mod))
y0.imps <- predict(mod0, model.frame(mod))
mean(y1.imps - y0.imps)
```

```
## [1] 0.61
```

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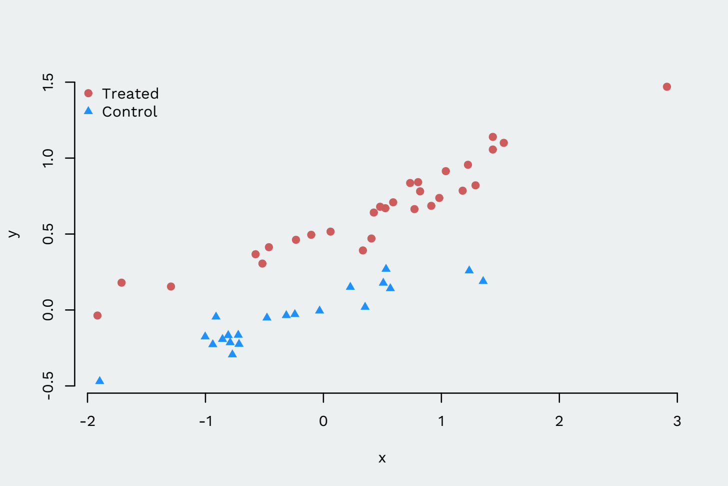
  - Kernel regression, local linear regression, regression trees, etc
  - Easiest is generalized additive models (GAMs)

Imputation estimator visualization



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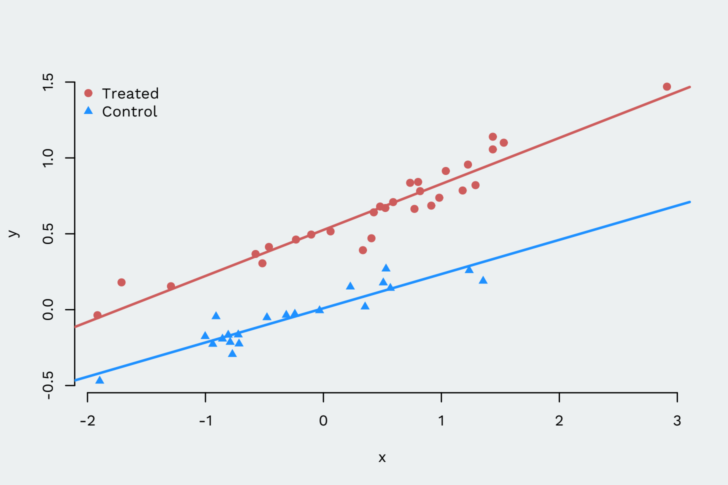

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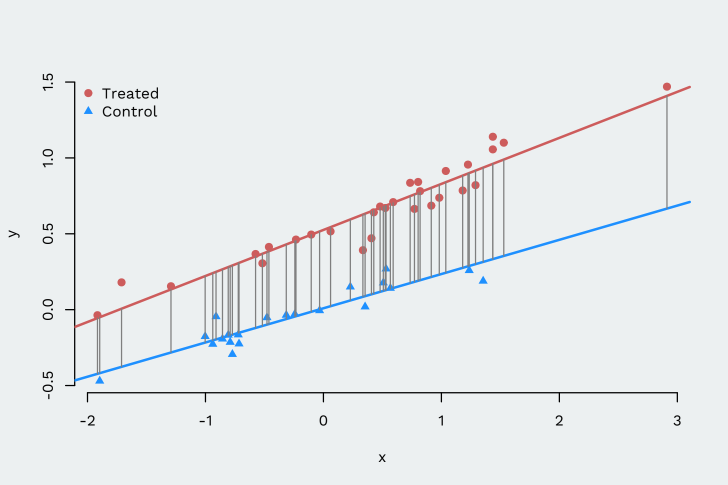
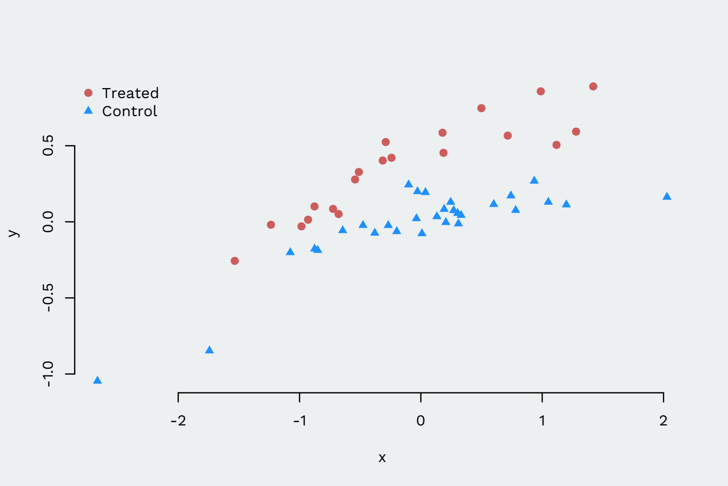


Nonlinear relationships

- Same idea but with nonlinear relationship between Y_i and X_i:



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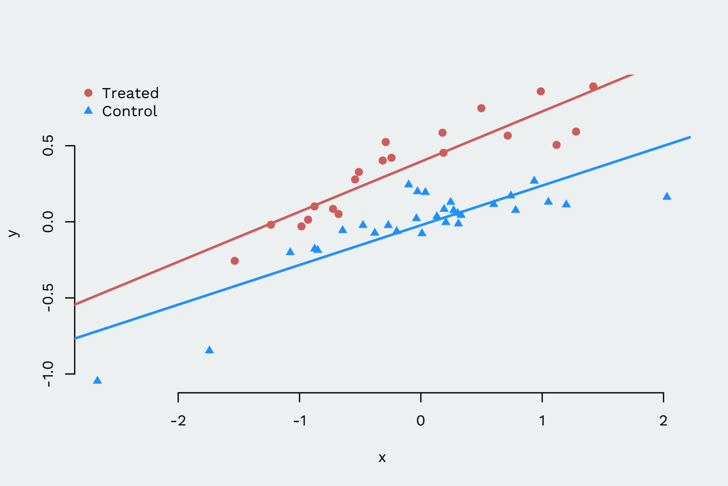

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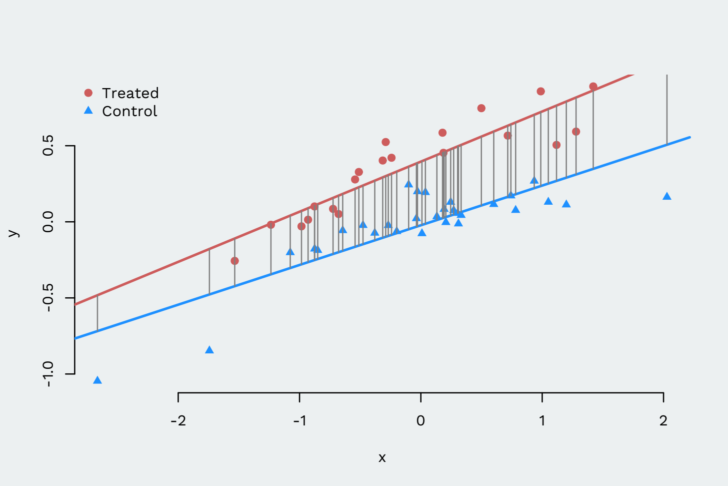

Using semiparametric regression

- Here, CEFs are nonlinear, but we don't know their form.

```r
library(mgcv)
mod0 <- gam(y ~ s(x), subset = d == 0)
summary(mod0)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## y ~ s(x)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.0225     0.0154   -1.46     0.16
##
## Approximate significance of smooth terms:
##        edf Ref.df    F p-value
## s(x) 6.03   7.08 41.3  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

Using semiparametric regression

- Here, CEFs are nonlinear, but we don't know their form.
- We can use GAMs from the `mgcv` package to for flexible estimate:

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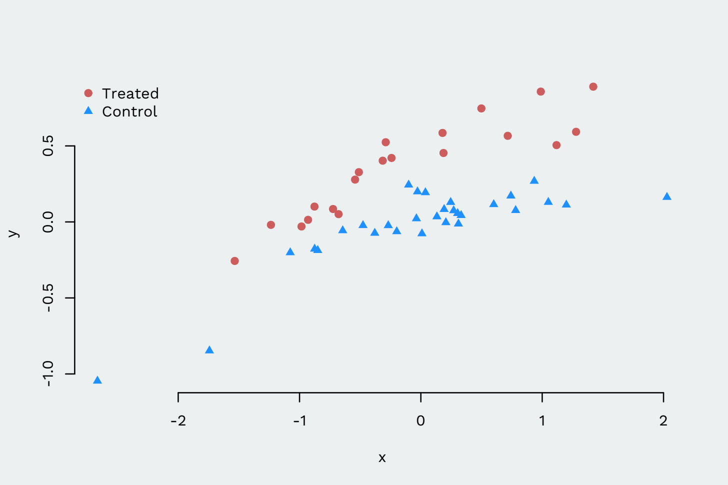

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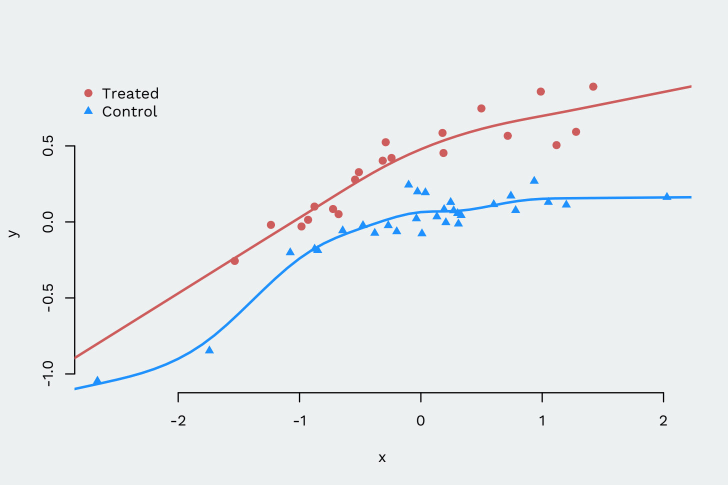

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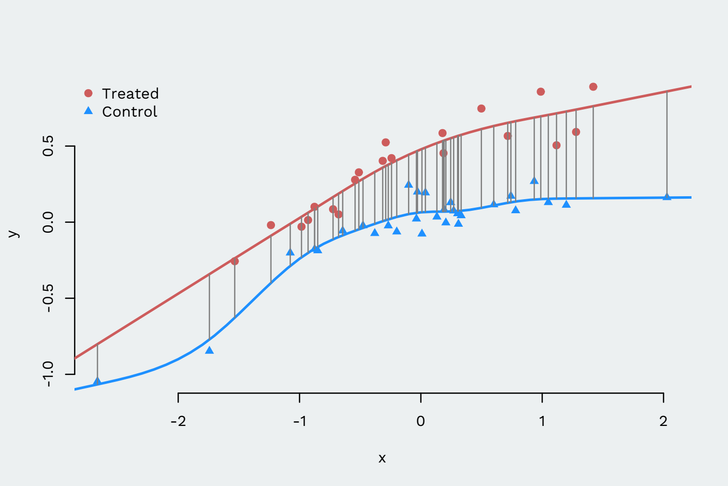

‘Wait...so what are we actually doing most of the time?’

A Discussion

Conclusions

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- Regression is mechanically very simple, but philosophically somewhat complicated
- It is a useful descriptive tool for approximating a conditional expectation function
- Once again though, the estimand of interest isn't necessarily the regression coefficient.
- There are many other approaches to estimation, but identification is key.

Next Week

- Causality with Unmeasured Confounding
- Reading:
  - Angrist and Pishke Chapter 4 Instrumental Variables and Chapter 6 on Regression Discontinuity Designs
  - Morgan and Winship Chapter 9 Instrumental Variable Estimators of Causal Effects
  - Optional: Hernan and Robins Chapter 16 Instrumental Variable Estimation

Visualization in the New York Times

AMERICAS

How Stable Are Democracies? ‘Warning Signs Are Flashing

The Interpreter

By AMANDA TAUB NOV. 29, 2016

WASHINGTON — Yascha Mounk is used to being the most pessimistic person in the room. Mr. Mounk, a lecturer in government at Harvard, has spent the past few years challenging one of the bedrock assumptions of Western politics: that once a country becomes a liberal democracy, it will stay that way.

His research suggests something quite different: that liberal democracies around the world may be at serious risk of decline.

Mr. Mounk’s interest in the topic began rather unusually. In 2014, he published a book, “Stranger in My Own Country.” It started as a memoir of his experiences growing up as a Jew in Germany, but became a broader investigation of how contemporary European nations were struggling to construct new, multicultural national identities.

Visualization in the New York Times

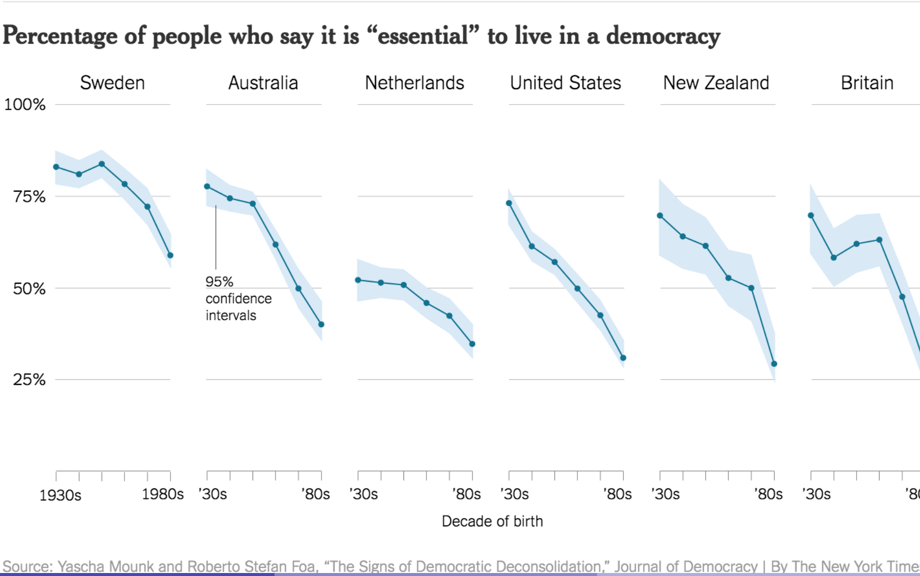
The Danger of Deconsolidation

THE DEMOCRATIC DISCONNECT

Roberto Stefan Foa and Yascha Mounk

***Roberto Stefan Foa** is a principal investigator of the World Values Survey and fellow of the Laboratory for Comparative Social Research. His writing has appeared in a wide range of journals, books, and publications by the UN, OECD, and World Bank. **Yascha Mounk** is a lecturer on political theory in Harvard University's Government Department and a Carnegie Fellow at New America, a Washington, D.C.–based think tank. His dissertation on the role of personal responsibility in contemporary politics and philosophy will be published by Harvard University Press, and his essays have appeared in* Foreign Affairs, *the* New York Times, *and the* Wall Street Journal.

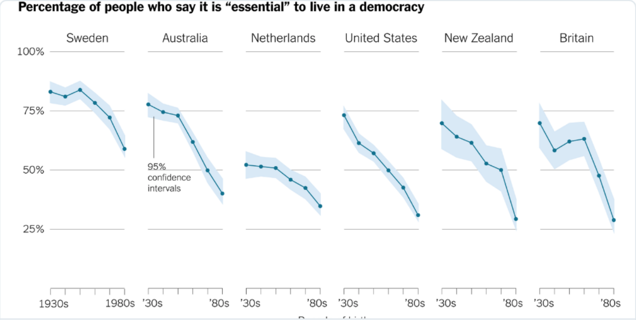
Visualization in the New York Times



Visualization in the New York Times

Ryan D. Enos @RyanDEnos · 19h

Lots of worried chatter a/b @amandataub article on work of @Yascha_Mounk. Important, but want to raise cautions 1/

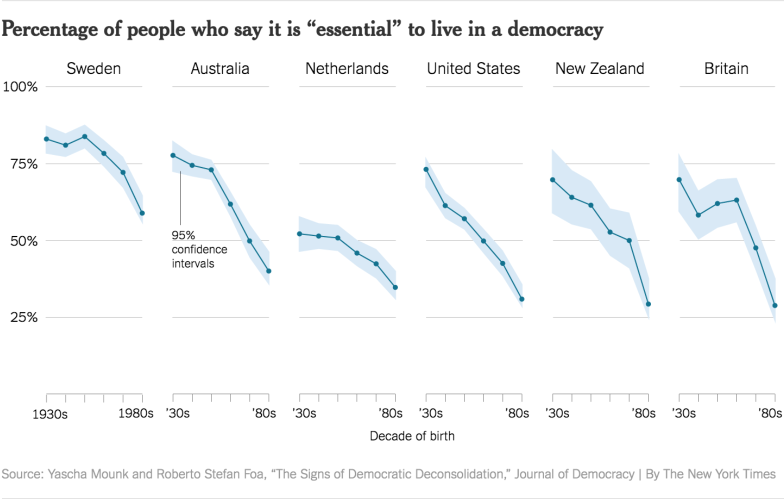


How Stable Are Democracies? 'Warning Signs Are Flashing Red'

New research tries to spot the collapse of liberal democracies before they happen, and it suggests that Western democracy may be seriously ill.

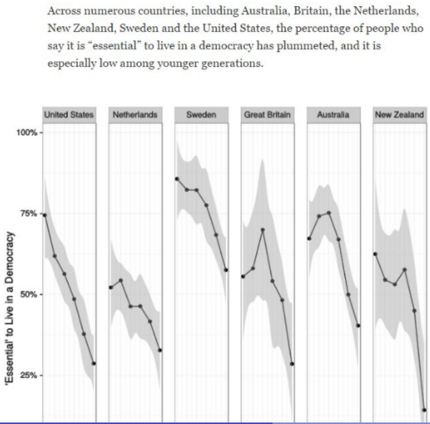
nytimes.com

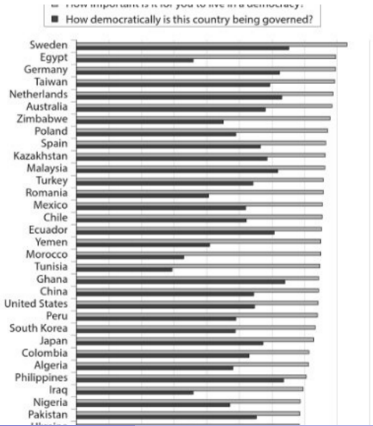
Alternate Graphs



Alternate Graphs

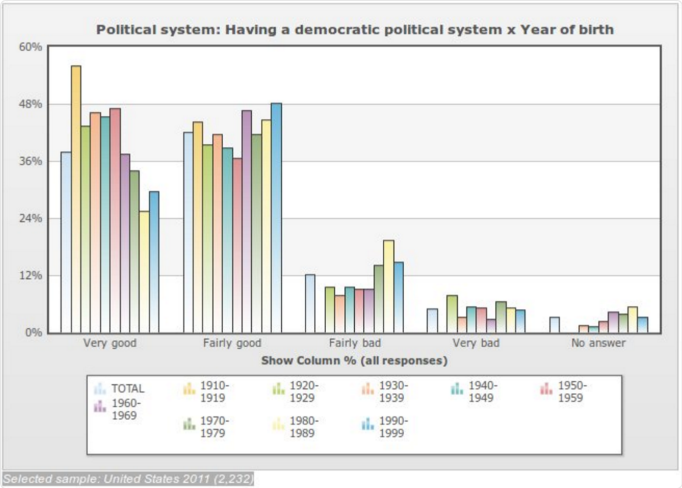
.@RyanDEnos Compare NYT/JoD (left) to the very same data analysed differently by Bartels and Achen (2016) (right). Extreme score vs means.





Alternate Graphs

@RyanDEnos They also stop at the 80s cohort. The data has the 90's as well. I wonder why they would stop there...



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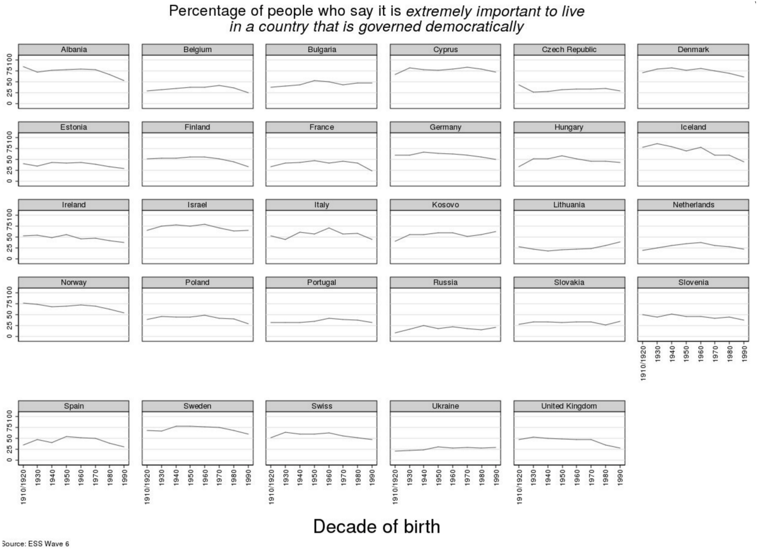
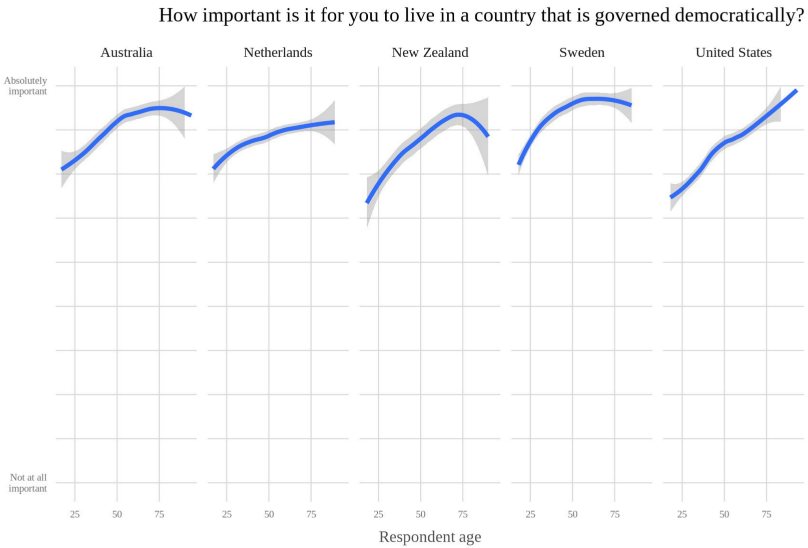


In reply to Ryan D. Enos

Benjamin Sack @bcsack · 15h

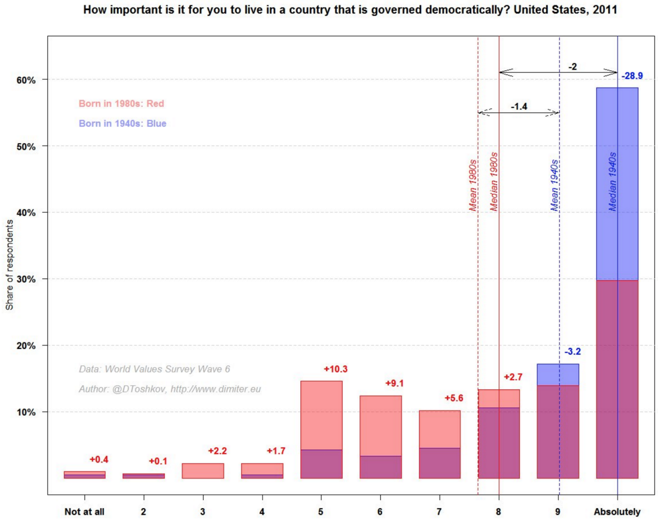
@RyanDEnos Same analysis strategy with comparable data from @ESS_Survey (similar item, 0-10 scale) shows slightly different pattern, too.

Alternate Graphs



614 Bantam @jpbach · 15h
@RyanDEnos @bshor @nataliemjb @TomWGvdMeer this is a "quick and dirty" plot I did with WVS wave 6. Not quite so terrifying.

Alternate Graphs



Dimiter Toshkov @DToshkov · 31m
my take on the democratic deconsolidation graph that scared everyone yesterday. Blue is 1940s cohort, red is 1980s. First, United States

Thoughts

Two stories here:

Thoughts

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1. Visualization and data coding choices are important

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Two stories here:

1. Visualization and data coding choices are important
2. The internet is amazing (especially with replication data being available!)