Precept 1: Probability, Simulations, Working with Data

Soc 500: Applied Social Statistics

Shay O'Brien

Princeton University

September 13, 2018

Support Resources

- Office hours
- Piazza
- Email
- Google is your best friend!

Office hours

Please raise your hand if you absolutely cannot make any of these office hours:

- Mondays 10am - 12pm (S + Z)
- Mondays 4:30 - 6:30pm (A + S + Z)
- Wednesdays 10am - 12pm (A + S + Z)
- Thursdays 1pm - 3pm (A + S)
- Thursdays 5pm - 7pm (A + Z)

Learning objectives

- Apply the Law of Total Probability and the Bayes' Rule

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Acknowledgements: These slides were most recently edited by Ziyao Tian, and they draw heavily on materials developed by past preceptors Shay O'Brien, Simone Zhang, Elisha Cohen, and Clark Bernier. Thanks!

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    - Have you done any R project before (R)?
- Let's find out:
    - What is the probability that a randomly selected person is taking Sociology course for the first time and has never done any R project before? $P(F, R^c)$?
    - Given that a person has done R programming project before, what is the probability that SOC400 is this person's first Sociology course? $P(F|R)$?

A Card Player's Example

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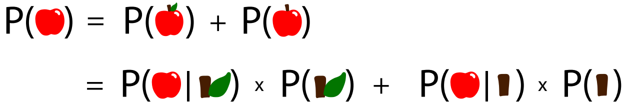
- $P(A, B) = P(A) \times P(B|A) = 4/52 \times 3/51 \approx .0045$

Law of Total Probability (LTP)

With 2 Events:

$$
\begin{aligned}
P(B) &= P(B, A) + P(B, A^c) \\
&= P(B|A) \times P(A) + P(B|A^c) \times P(A^c)
\end{aligned}
$$



Recall, if we randomly draw two cards from a standard 52 card deck and define the events $A = \{\text{Ace on Draw 1}\}$ and $B = \{\text{Ace on Draw 2}\}$, then

- $P(A) = 4/52$
- $P(B|A) = 3/51$
- $P(A, B) = P(A) \times P(B|A) = 4/52 \times 3/51$

Question: $P(B) = ?$

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$$\begin{aligned} P(B) &= 3/51 \times 1/13 + 4/51 \times 12/13 \\ &= \frac{3+48}{51 \times 13} = \frac{1}{13} = \frac{4}{52} \end{aligned}$$

Example: Voter Mobilization

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=& .51
\end{aligned}$$

Prosecutor's Fallacy

A woman has been murdered, and her husband is accused of having committed the murder. It is known that the man abused his wife repeatedly in the past, and the prosecution argues that this is important evidence pointing towards the man's guilt. The defense attorney says that the history of abuse is irrelevant, as only 1 in 1000 women who experience spousal abuse are subsequently murdered.

Assume that the defense attorney's 1 in 1000 figure is correct, and that half of men who murder their wives previously abused them. Also assume that 20% of murdered women were killed by their husbands, and that if a woman is murdered and the husband is not guilty, then there is only a 10% chance that the husband abused her. What is the probability that the man is guilty? Is the prosecution right that the abuse is important evidence in favor of guilt?

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  M => woman is murdered
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- What do we know?

Prosecutor's Fallacy

Table: What Do We Know?

Statements	We know	We also know
1 in 1000 women who experience spousal abuse are subsequently murdered	$P(?\|?) = 0.001$	$P(?\|?) = 1 - 0.001$
half of men who murder their wives previously abused them	$P(?\|?) = 0.5$	$P(?\|?) = 1 - 0.5$
20% of murdered women were killed by their husbands	$P(?\|?) = 0.2$	$P(?\|?) = 1 - 0.2$
woman is murdered and the husband is not guilty, then there is only a 10% chance that the husband abused her	$P(?\|?) = 0.1$	$P(?\|?) = 1 - 0.1$

Prosecutor's Fallacy

Table: What Do We Know?

Statements	We know	We also know
1 in 1000 women who experience spousal abuse are subsequently murdered	$P(M\|A) = 0.001$	$P(M^c\|A) = 1 - 0.001$
half of men who murder their wives previously abused them	$P(A\|M, G) = 0.5$	$P(A^c\|M, G) = 1 - 0.5$
20% of murdered women were killed by their husbands	$P(G\|M) = 0.2$	$P(G^c\|M) = 1 - 0.2$
woman is murdered and the husband is not guilty, then there is only a 10% chance that the husband abused her	$P(A\|G^c, M) = 0.1$	$P(A^c\|G^c, M)$=1-0.1

Prosecutor's Fallacy

- What we know $P(M|A)$, $P(A|M,G)$, $P(G|M)$, $P(A|G^c,M)$

Prosecutor's Fallacy

- What we know $P(M|A)$, $P(A|M, G)$, $P(G|M)$, $P(A|G^c, M)$
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Bayes' Rule

- Often we have information about $P(B|A)$, but require $P(A|B)$ instead.
- When this happens, always think **Bayes' Rule**
- Bayes' rule: if $P(B) > 0$

$$\mathrm{P(A \mid B)} = \frac{P(B|A)P(A)}{P(B)}$$

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$$\mathrm{P(A \mid B)} = \frac{P(B|A)P(A)}{P(B)}$$

- Also recall from the definition of conditional probability:

$$\mathrm{P(A, B) = P(B \mid A)P(A)}$$

Prosecutor's Fallacy

$$P(M|A) = 1/1000$$
$$P(A|G, M) = 1/2$$
$$P(G|M) = 1/5$$
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How do we find P(A | M)?

Recall Law of Total Probability:

$$P(X) = P(X|Y)P(Y) + P(X|Y^c)P(Y^c)$$

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Applying here:

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Putting it all together:

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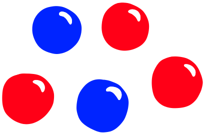
Prosecutor's Fallacy

- What does this mean for our defendant?

R Markdown

- install.packages("knitr")
- File - New File - R Markdown
- Preferences - Under Sweave set "Weave Rnw files with" to "knitr"
- See 1_Sample Markdown Document.Rmd

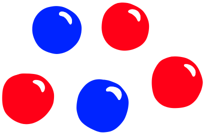
Probability by Simulation



Problem: You have a bag of five marbles. Three are red and two are blue. You draw one marble. Without replacing it, you then draw another marble.
What is the probability that the two marbles are the same color?

- We could do this analytically:

Probability by Simulation



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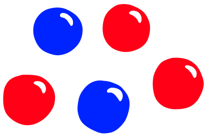
  $=P(D1 = R)P(D2 = R \mid D1 = R) + P(D1 = B)P(D2 = B \mid D1 = B)$
  $= (3/5)(2/4) + (2/5)(1/4)$
  $= 2/5$

Probability by Simulation



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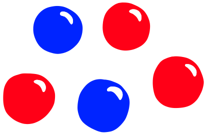
  $= (3/5)(2/4) + (2/5)(1/4)$
  $= 2/5$
- Or we can run a simulation!
  See 2_Simulation example.R

Writing Functions



- We've already used many built in R functions: `mean()`, `head()`, etc.
- We can also define our own functions:

Define a function that takes 3 arguments; it will add the first two and divide by the third:

```
> myfunction <- function(x, y, z){
+   out <- (x + y) / z
+   return(out)
+ }
> ## use the function
> myfunction(1, 5, 2)

[1] 3
```

Data Manipulation and Tables

See 3_Data Manipulations and Tables.Rmd

Fun with Lint!!

adv-r.had.co.nz/Style.html

RStudio → Preferences →
Code → Diagnostics →
"Provide R style diagnostics"

Resources

- R Style Guide: `http://adv-r.had.co.nz/Style.html`
- R Cookbook: `http://www.cookbook-r.com/`
- ggplot cheatsheet: `https://www.rstudio.com/wp-content/uploads/2015/08/ggplot2-cheatsheet.pdf`
- dplyr cheatsheet: `https://www.rstudio.com/wp-content/uploads/2015/02/data-wrangling-cheatsheet.pdf`
- Probability cheatsheet: `https://static1.squarespace.com/static/54bf3241e4b0f0d81bf7ff36/t/55e9494fe4b011aed10e48e5/1441352015658/probability_cheatsheet.pdf`
- Probability and statistics visualizations: `http://students.brown.edu/seeing-theory/index.html`
- Kosuke Imai's textbook contains lots of sample R code!