Precept 8: Some review, heteroskedasticity, and causal inference

Soc 500: Applied Social Statistics

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Learning Objectives

1. Review
    1. Calculating error variance
    2. Interaction terms (common support, main effects)
    3. Model interpretation ("increase", "intuitively")
    4. Heteroskedasticity
2. Causal inference with potential outcomes

[0] Thanks to Ian Lundberg and Xinyi Duan for material and ideas.

Calculating error variance

- We have some data: Y, X, Z.
- We think the correct model is $Y = X + Z + u$.
- We estimate this conditional expectation using OLS:
  $Y = \beta_0 + \beta_1 X + \beta_2 Z$
- We want to know the standard error of β_1.

Standard error of β_1

$SE(\hat{\beta}_j) = \sqrt{\frac{1}{1-R_j^2} \frac{\hat{\sigma_u^2}}{\sum_{i=1}^n (x_{ij} - \bar{x}_j)^2}}$, where R_j^2 is the R^2 of a regression of variable j on all others.

Question: What is $\hat{\sigma_u^2}$?

Calculating error variance

$$\hat{\sigma_u^2} = \frac{\sum_i \hat{u_i}^2}{DF_{resid}}$$

- You can adjust this in finite samples by $\hat{\bar{u}}$ (why?)

Interaction terms

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$$

- Assume $X \sim \mathcal{N}(?, ?)$ and $Z \in \{0, 1\}$

Interaction terms

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$$

Scenario 1

- When $Z = 0$, $X \sim \mathcal{N}(3, 4)$
- When $Z = 1$, $X \sim \mathcal{N}(-3, 2)$
- Do you think an interaction term is justified here?

Interaction terms

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$$

Scenario 2

- When $Z = 0$, $X \sim \mathcal{N}(0, 10)$
- Let $D \sim \text{Bern}(0.5)$
- When $Z = 1$ and $D = 0$, $X \sim \mathcal{N}(-3.1, 4)$
- When $Z = 1$ and $D = 1$, $X \sim \mathcal{N}(2.2, 5)$
- Do you think an interaction term is justified here?

Interaction terms

$$Y = \beta_0 + \beta_1 X + \beta_3 XZ$$

- What are we now assuming about the true relationship?

Interaction terms

$$Y = \beta_0 + \beta_1 X + \beta_3 XZ$$

- When $X = 0$, $E[Y|Z = 0] = E[Y|Z = 1]$
- $Cov[u, Z] = 0$ (why?)

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- "On average, we would expect units with 1 percentage point higher X to have 12 percentage points higher Y."

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- **HARK**ing: "**H**ypothesizing **A**fter **R**esults are **K**nown"

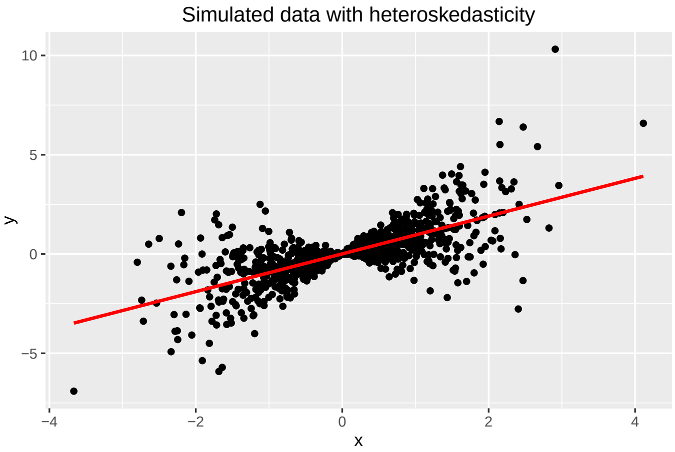
Residual plots for homoskedasticity

- If homoskedasticity is violated, the spread of the residuals *around the conditional mean* will vary with the predicted values.

Violation of homoskedasticity

I generated some data that violates homoskedasticity (the error variance depends on X):

$$Y = X + u, u \sim N(0, |X|)$$

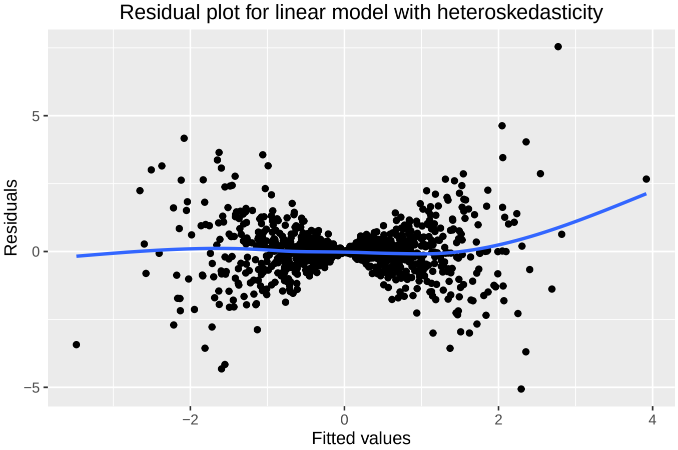


Violation of homoskedasticity

Then I fit a linear model.

$$Y = \beta_0 + \beta_1 X + v$$

The residual plot is below. The error variance is related to the fitted values - homoskedasticity is violated!



Exploring causal inference with a running example

Research question

Does college education cause higher earnings?

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Does college education cause higher earnings?

Potential (internal) problems:

- Fundamental problem: We cannot observe both states
- Selection bias: Higher ability people may select into college
- SUTVA violation: Your sister's college education might affect your earnings, or "college" might be multiple things
- Heterogeneity: Different effects on different people

Neyman-Rubin Model: Potential outcomes

(from lecture slides)

Two possible conditions:

- Treatment condition $T = 1$
- Control condition $T = 0$

Suppose that we have an individual i.

Key assumption: we can imagine a world where individual i is assigned to treatment and control conditions

Potential Outcomes: responses under each condition, $Y_i(T)$

- Response under treatment $Y_i(1)$
- Response under control $Y_i(0)$

Potential outcomes in our college-earnings example

In our example examining how college affects future earnings, what do the numbers in the table below represent?

	Treatment ($Y_i(1)$)	Control ($Y_i(0)$)
Person 1	45,000	32,000
Person 2	54,000	45,000
Person 3	34,000	34,000

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Why can we never estimate τ_i?

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- The individual causal effect is $\tau_i = Y_i(1) - Y_i(0)$

Why can we never estimate τ_i? Because we can never observe an individual in both the treatment and control states.
This is the fundamental problem of causal inference.

Fundamental Problem of Causal Inference

Holland 1986: Only one outcome can be observed

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Causal inference with a strong assumption

$$T \rightarrow Y$$

- T is the treatment (college education)
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To make causal inferences, we must assume ignorability:

$$\{Y_i(1), Y_i(0)\} \perp\!\!\!\perp T_i$$

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Meaning, if college education is ignorable with respect to potential earnings, then the average causal effect of college on earnings is just the average earnings difference between the two groups.

Identification and the role of assumptions

- A causal effect is identified if we could pin it down with an infinite amount of data.
- In any causal study we must state the assumptions under which a causal effect is identified.
- Law of Decreasing Credibility (Manski): The credibility of inference decreases with the strength of the assumptions maintained

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Assuming ignorability, the causal effect of college on earnings is identified by the mean difference between the two groups.
But this is a heroic assumption, so inference is not very credible!

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In our college-earnings example, what would violate each of these assumptions?

1. Elite college (Princeton) might be a different treatment from public universities. Only one treatment allowed.
2. My sister's college education might affect my earnings. Interference not allowed.

Definitions of treatment effects

Average treatment effect (ATE): Average over the whole population

$$E[Y_i(1) - Y_i(0)]$$

Average treatment effect on the treated (ATT): Average among those who take the treatment

$$E[Y_i(1) - Y_i(0) \mid D_i = 1]$$

Average treatment effect on the control (ATC): Average among those who do not take the treatment

$$E[Y_i(1) - Y_i(0) \mid D_i = 0]$$

We could define an average treatment effect for any subpopulation of interest.

Definitions of treatment effects

In the case of college education and earnings,

- the ATT is the effect of college among those who attend
- the ATC is the effect among those who do not attend
- the ATE is the average effect over the whole population

In what situations might we care about each one?

Immutable characteristics

from lecture slides

There are three problems with race as a treatment in the causal inference sense

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3. Race is unstable
   - there is substantial variance across treatments which is a SUTVA violation

Objections to potential outcomes

(Morgan and Winship concluding chapter)

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- Appropriate for effects of causes, not causes of effects
  - but we have to start with small steps before reaching for grand claims
- Relies on metaphysical quantities that cannot be observed
  - but we try to get at these with as minimal assumptions as possible
- People rarely have stable characteristics across treatment thresholds
  - e.g. graduate students, vampires; see Paul & Healy (2014), "Transformative Treatments"