Week 4: Testing/Regression

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Princeton

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[1]These slides are heavily influenced by Matt Blackwell, Adam Glynn, Jens Hainmueller, and Erin Hartman

Where We've Been and Where We're Going...

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- Last Week
  - inference and estimator properties
  - point estimates, confidence intervals

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  - properties of ordinary least squares

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- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causal inference

1. Hypothesis Testing
   - Terminology and Procedure
   - One-Sided Tests
   - Connections
   - Power
2. p-values
   - Mechanics
   - Multiple Testing
   - Fun With Salmon
   - The Significance of Significance
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   - Conditional Expectation Functions
   - Nonparametric Regression
   - Best Linear Predictor
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We Secretly Already Covered This!

American Political Science Review Vol. 102, No. 1 February 2008
DOI: 10.1017/S000305540808009X

Social Pressure and Voter Turnout: Evidence from a Large-Scale Field Experiment

ALAN S. GERBER *Yale University*
DONALD P. GREEN *Yale University*
CHRISTOPHER W. LARIMER *University of Northern Iowa*

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Dear Registered Voter:

WHAT IF YOUR NEIGHBORS KNEW WHETHER YOU VOTED?

Why do so many people fail to vote? We've been talking about the problem for years, but it only seems to get worse. This year, we're taking a new approach. We're sending this mailing to you and your neighbors to publicize who does and does not vote.

The chart shows the names of some of your neighbors, showing which have voted in the past. After the August 8 election, we intend to mail an updated chart. You and your neighbors will all know who voted and who did not.

DO YOUR CIVIC DUTY—VOTE!

MAPLE DR	Aug 04	Nov 04	Aug 06
9995 JOSEPH JAMES SMITH	Voted	Voted	______
9995 JENNIFER KAY SMITH		Voted	______
9997 RICHARD B JACKSON		Voted	______
9999 KATHY MARIE JACKSON		Voted	______

Review of the Gerber, Green and Larimer Result

- Our estimand is the difference in population means: $\theta = \mu_y - \mu_x$. Where μ_y is those receiving the social pressure mailer and μ_x is those receiving the civic duty mailer.

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- This implies that $\hat{\theta} \sim \mathcal{N}(0, \text{SE}(\hat{\theta}))$

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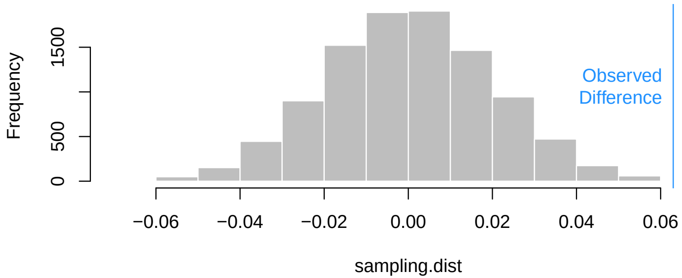
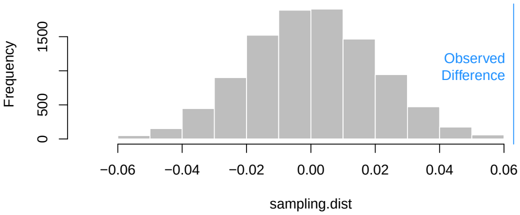
Example from Gerber, Green and Larimer (2008).



Overall Idea



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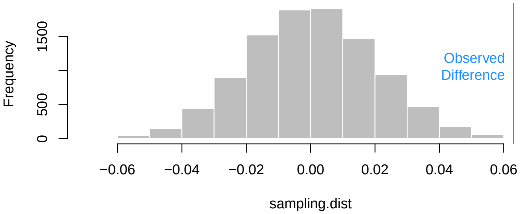

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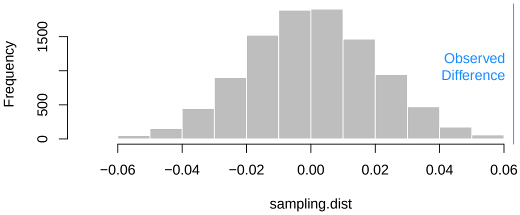

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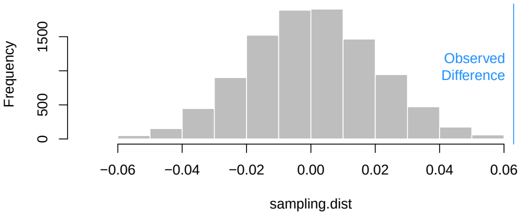

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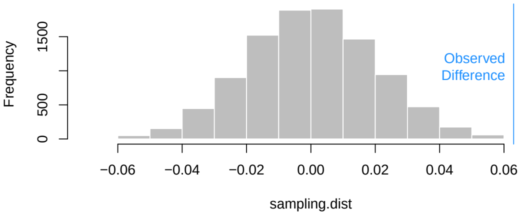

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- We derived the sampling distribution under that value.
- We asked 'if the population difference was zero, how likely would it be to see an observe difference as extreme (or more extreme) than our observed estimate?'
- Our observed difference was so implausible we concluded it was unlikely the population difference was really zero.

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- Hypothesis tests lead to discrete decisions.

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- The trial included 345 patients with initial systolic blood pressure between 140-159.
- Each subject was assigned to take the drug combination for 16 weeks.
- Systolic blood pressure was measured on each subject before and after the treatment period.

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Subject	SBP_{before}	SBP_{after}	Decrease
1			
2			
3			
4			
⋮			
345			

Example

Subject	SBP_{before}	SBP_{after}	Decrease
1	147		
2	153		
3	142		
4	141		
⋮	⋮		
345	155		

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4	141	134	7
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345	155	115	40

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Question: Should the FDA allow the drug to proceed to the next stage of testing?

The FDA's Decision

We can think of the FDA's problem in terms of two dimensions:

- The true state of the world
- The decision made by the FDA

	Drug works	Drug doesn't work
FDA approves		
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Two kinds of bad decisions:

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- Test Level (the probability of a type I error)
- Rejection Region (the basis of our decision)

Hypotheses

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  Example: There is not a difference in voting rates between those who received the social pressure mailer and those that received the civic duty mailer.

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- Alternative Hypothesis (H_a): The state of the world where the null hypothesis is not true and thus the claim to be indirectly tested.

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Error Types

	(H_0 False)	(H_0 True)
Reject H_0		
Don't Reject H_0		

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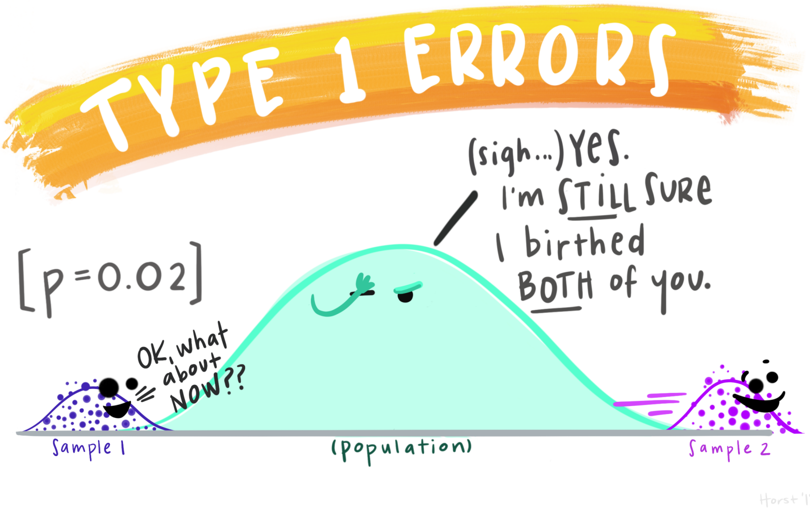
We generally make the normative judgment that we prefer an undetected finding (Type II error) to a false discovery (Type I error).

A Visual Reminder from Allison Horst



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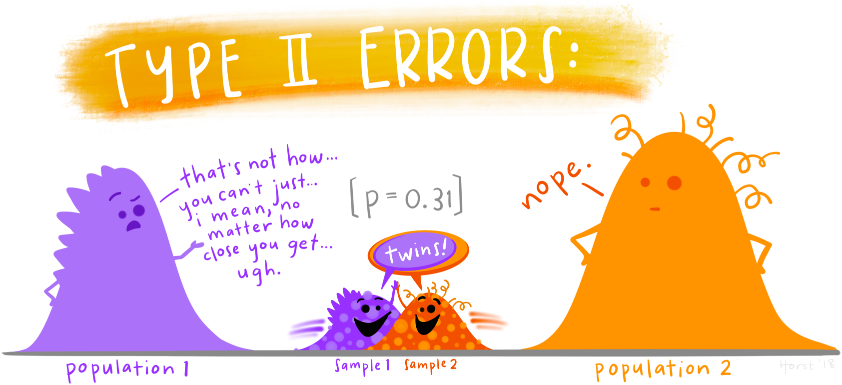

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Test Statistics and Null Distributions

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Test Statistic: which we denote T_n is a function of the sample, the estimator and the null hypothesis.

$$T_n = \frac{\hat{\theta} - \theta_0}{\widehat{\mathsf{SE}}[\hat{\theta}]}$$

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Null Distribution: the sampling distribution of the statistic/test statistic assuming that the null is true.

Test Statistics and Null Distribution

Returning to the voting experiment. We know from the Central Limit Theorem that the standardized difference in means has a standard normal distribution asymptotically,

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Under the null hypothesis of $\mu_y - \mu_x = 0$, we have

$$T_n = \frac{\hat{\theta}}{\widehat{\text{SE}}[\hat{\theta}]} \xrightarrow{d} \mathcal{N}(0,1)$$

If T_n is very far from zero—in the sense that it has low probability under $\mathcal{N}(0,1)$—then we reject the null hypothesis as not plausible.

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- We call c the critical value.
- Much like the 95% confidence interval, we pick $\alpha = .05$ by convention.

The value for which P=0.05, or 1 in 20, is 1.96 or nearly 2; it is convenient to take this point as a limit in judging whether a deviation ought to be considered significant or not. Deviations exceeding twice the standard deviation are thus formally regarded as significant. Using this criterion we should be led to follow up a false indication only once in 22 trials, even if the statistics were the only guide available. Small effects will still escape notice if the data are insufficiently numerous to bring them out, but no lowering of the standard of significance would meet this difficulty.

- Ronald Fisher, *Design of Experiments* (1922)

Two-sided Rejection Region

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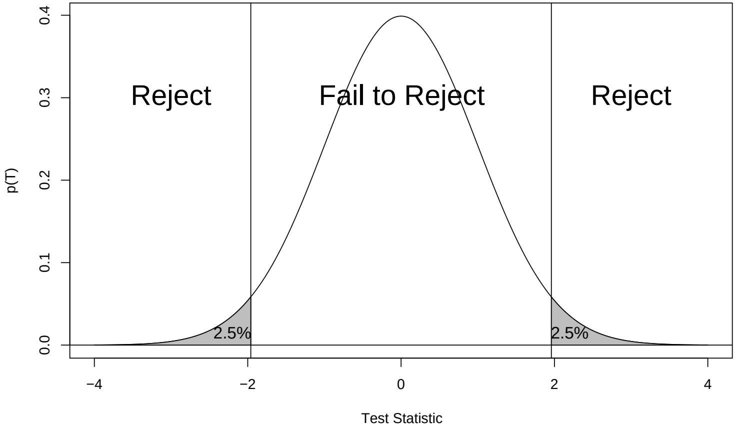
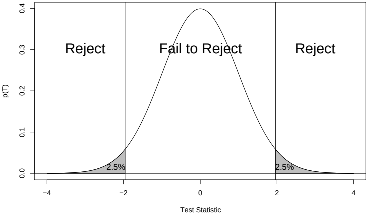
Rejection region with $\alpha = .05$, $H_0 : \theta = 0$, $H_A : \theta \neq 0$:



Defining the Rejection Region



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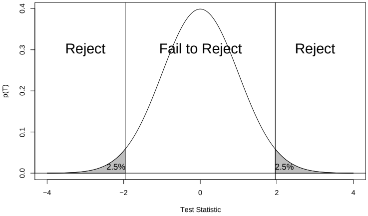

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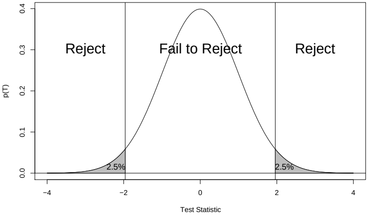

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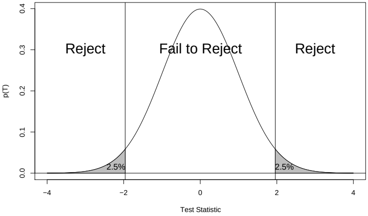

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- This is just the task of finding the quantile for $\alpha/2$. In the case of $\alpha = .05$, $\texttt{qnorm}(.05/2) = 1.96$

The Complete Recipe

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2. Choose a test statistic T_n.

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6. If your observed data is sufficiently unlikely under your null hypothesis, reject your null.

The Gerber, Green and Larimer Example

```
diff <- mean(treated) - mean(control)
se_diff <- sqrt(var(treated)/length(treated) +
                var(control)/length(control))
test_statistic <- diff/se_diff
```

This yields 18.3 which is much better than our .05 critical value of 1.96.

We reject the null.

Back to the FDA

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Drug trials are expensive and *ex ante* we can specify that we only care about one direction in particular. Consider the Sowers et al (2006) case which claimed to decrease blood pressure.

Sowers et al. Example

We can define our hypotheses for a one-sided test.

$$H_0 : \mu_{decrease} \leq 0 \quad (1)$$
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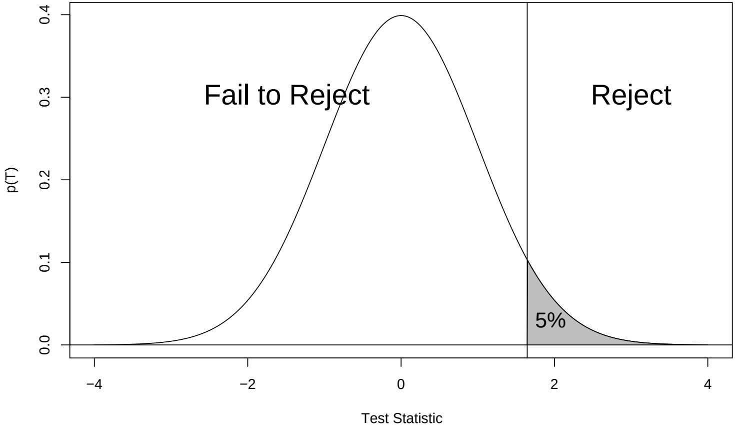
Therefore,

$$t_n = \frac{21.0 - 0}{\frac{14.3}{\sqrt{345}}} = 27.3$$

We construct our rejection region with $c = \texttt{qnorm(.95)} = 1.644$.

Rejection Region with $\alpha = .05$



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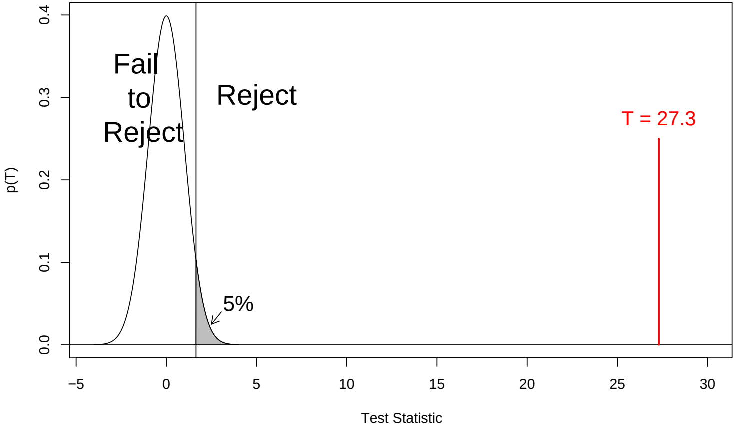

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- This will guarantee the nominal probability of Type I error as n gets large.

Connections to Confidence Intervals

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We can think of confidence intervals as a range of plausible values in the sense that we would not have rejected them had they been our null hypotheses.

1. Hypothesis Testing
   - Terminology and Procedure
   - One-Sided Tests
   - Connections
   - Power
2. p-values
   - Mechanics
   - Multiple Testing
   - Fun With Salmon
   - The Significance of Significance
3. What is Regression?
   - Conditional Expectation Functions
   - Nonparametric Regression
   - Best Linear Predictor
   - Ordinary Least Squares
4. Interpreting Regression
   - Fun With Linearity

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3. Null is false, the test is well-powered and we got incredibly unlucky.

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- This might seem obvious but conflating a lack of evidence with evidence for a zero effect is a problem that crops up in a lot of work.
- Power analysis is a way of guiding the choice of sample size prior to an experiment to avoid this kind of mistake.

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5) Possibly repeat under different assumptions about the population.

Example: Power Analysis

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TABLE 2. Effects of Four Mail Treatments on Voter Turnout in the August 2006 Primary Election

	Experimental Group				
	Control	Civic Duty	Hawthorne	Self	Neighbors
Percentage Voting	29.7%	31.5%	32.2%	34.5%	37.8%
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- If their results were exactly true, would we be expect to be able to confirm that the effect is non-zero with a replication using only 500 mailers?
- For this example, let's ignore the household sampling, but see the challenge problem from Course Meeting 3.

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- Using the sampling distribution we derived can calculate:

$$P\left(\hat{\theta}_n < -1.96\sqrt{.001804}\right) + P\left(\hat{\theta}_n > 1.96\sqrt{.001804}\right)$$

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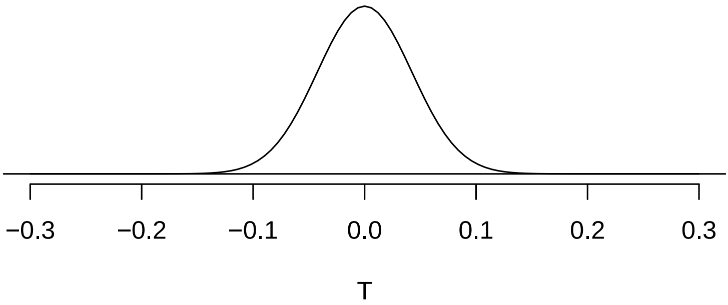
Yikes! That is not well powered.

Power Graph



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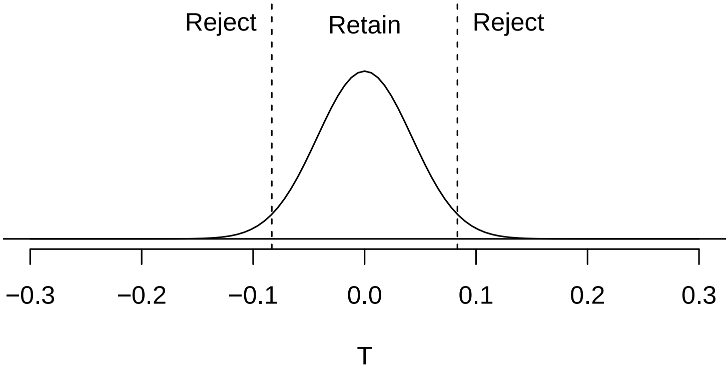

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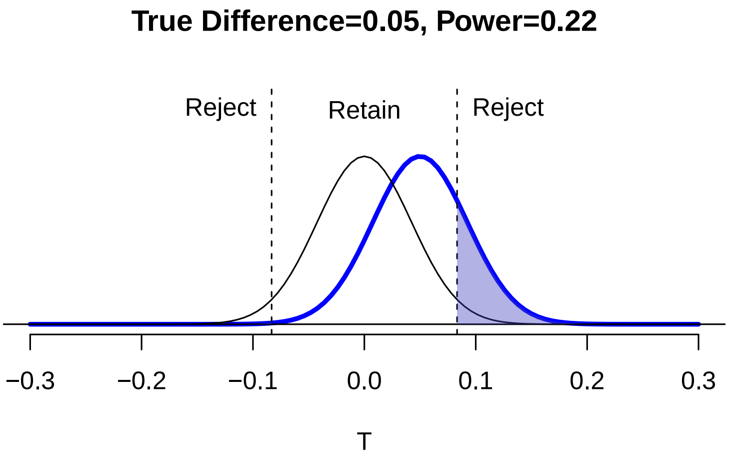
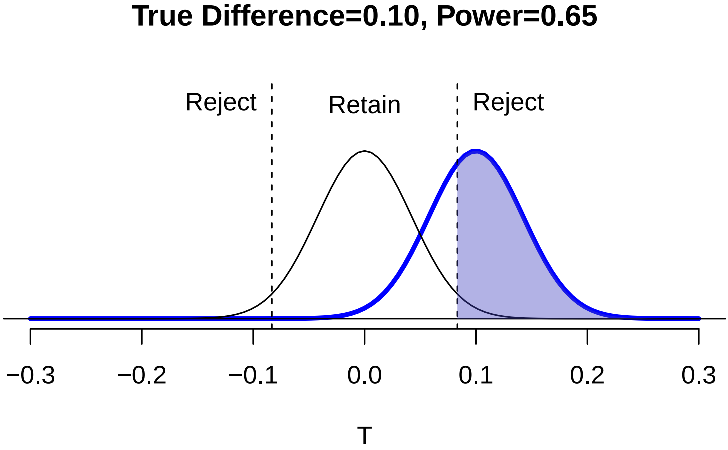


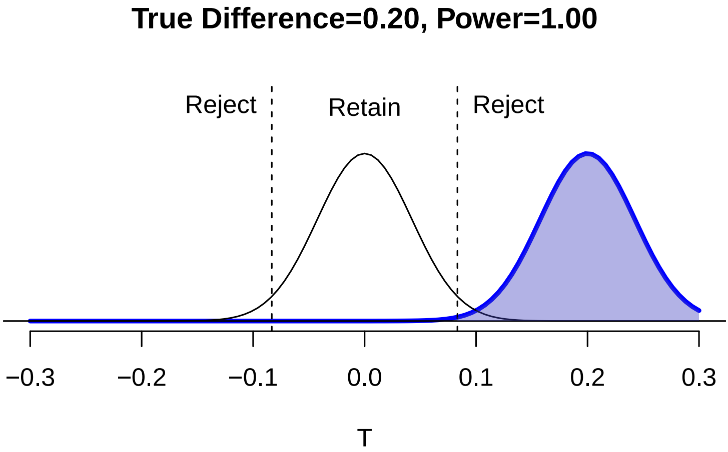
True Difference=0.05, Power=0.22
Reject
Retain
Reject
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
T

Power Graph



True Difference=0.10, Power=0.65
Reject
Retain
Reject
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
T

Power Graph


True Difference=0.20, Power=1.00
Reject
Retain
Reject
−0.3
−0.2
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0.0
0.1
0.2
0.3
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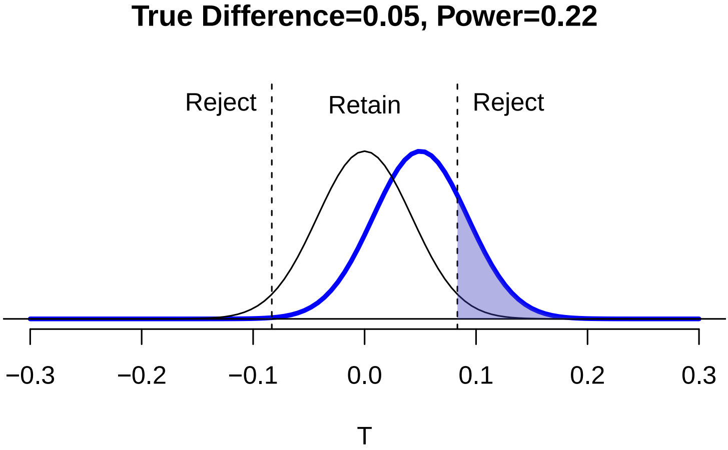

True Difference=0.05, Power=0.22
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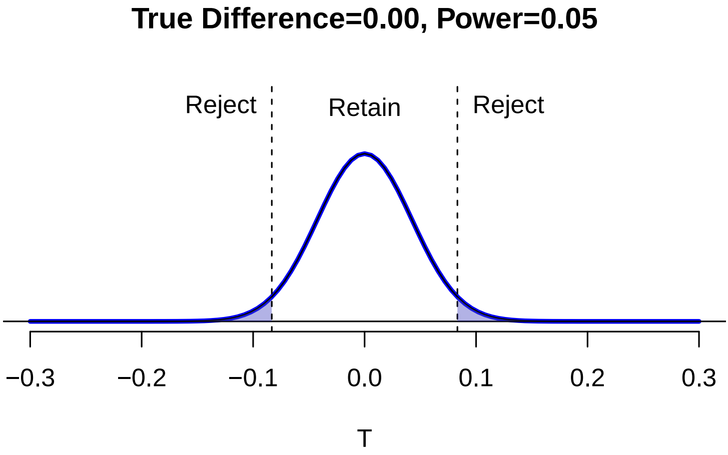
True Difference=0.00, Power=0.05
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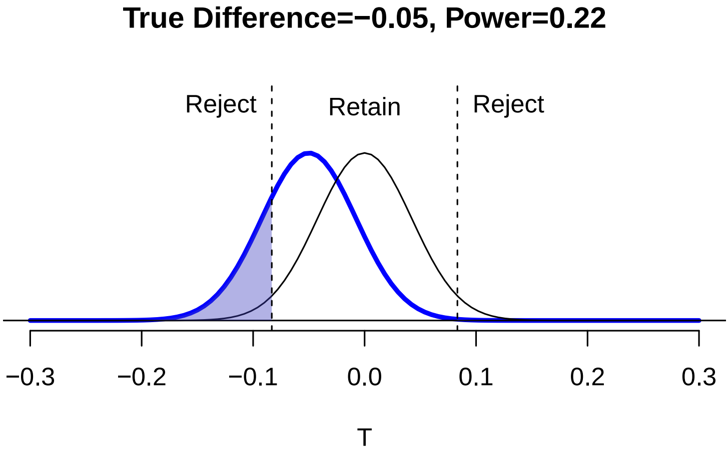

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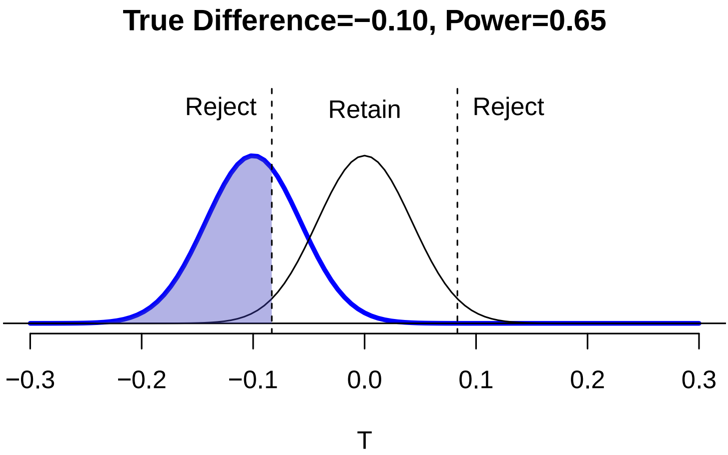

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0.3
T

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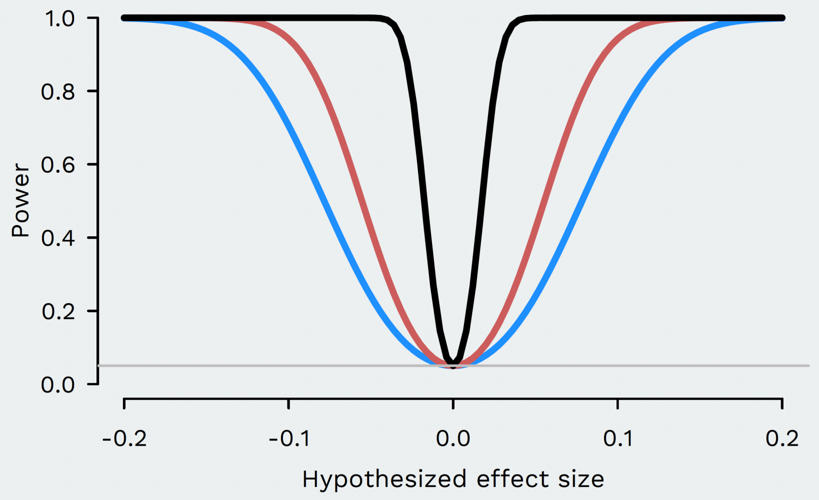
Yeah. . . kind of. In practice, this is why we calculate under many possible configurations.

Power Curve

You can graph power for various possible effect sizes (here for 500, 1000, and 10000 samples).



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  - bigger difference (pushes the alternative distribution away)
  - smaller variance (it squeezes the distribution in)
- Power analysis is really important if you are planning experiments, but we will touch on it only cursorily in this class. The Gerber and Green Field Experiments book is an amazing resource for more on experiments in general.

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Next time: *p*-values!

Where We've Been and Where We're Going...

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- Last Week
  - inference and estimator properties
  - point estimates, confidence intervals
- This Week
  - hypothesis testing
  - what is regression?
  - nonparametric and linear regression
- Next Week
  - inference for simple regression
  - properties of ordinary least squares
- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causal inference

1 Hypothesis Testing
- Terminology and Procedure
- One-Sided Tests
- Connections
- Power

2 p-values
- Mechanics
- Multiple Testing
- Fun With Salmon
- The Significance of Significance

3 What is Regression?
- Conditional Expectation Functions
- Nonparametric Regression
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Under the null hypothesis, this corresponds to the probability of observing a test statistic as extreme or more extreme than the one in the observed data (where extreme is defined in terms of the alternative hypothesis).

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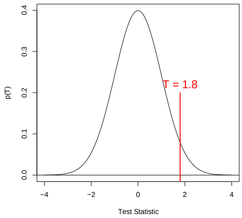

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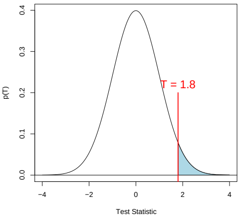

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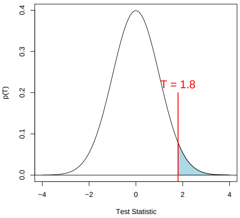

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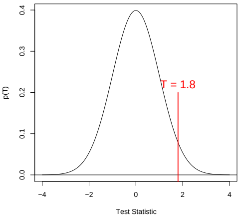

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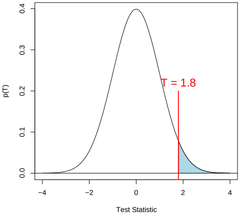

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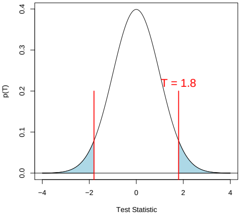

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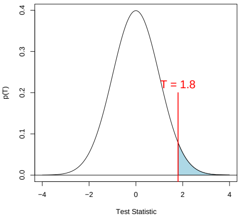

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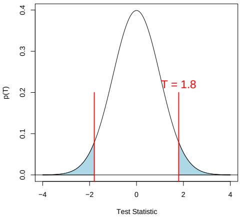

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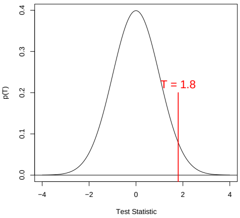

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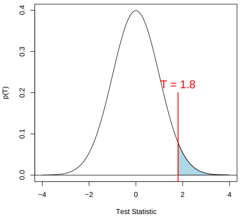

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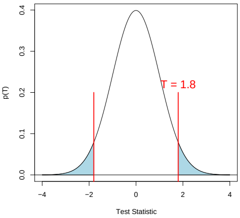

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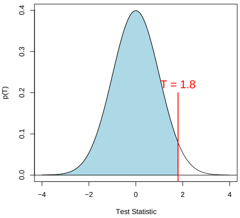

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Rejection Regions and p-values

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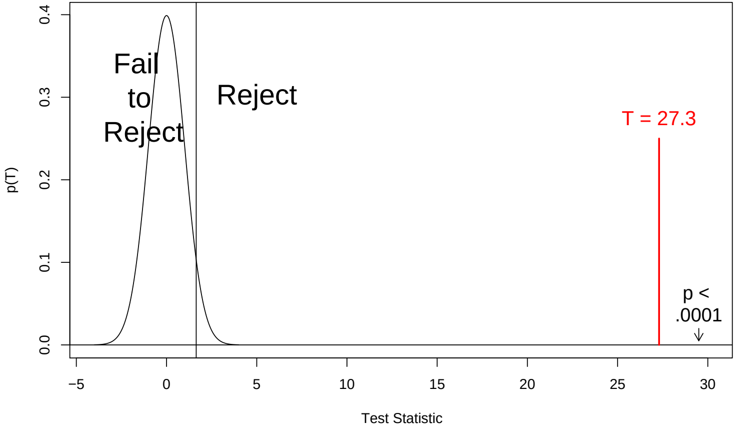
- We can get this in R with $2 * \texttt{pnorm}(-18.5)$
- That yields a p-value of 2.06×10^{-76}.
- By convention we would say it is statistically significant at level α for some α that the p-value is below.

The Sowers et. al. Example



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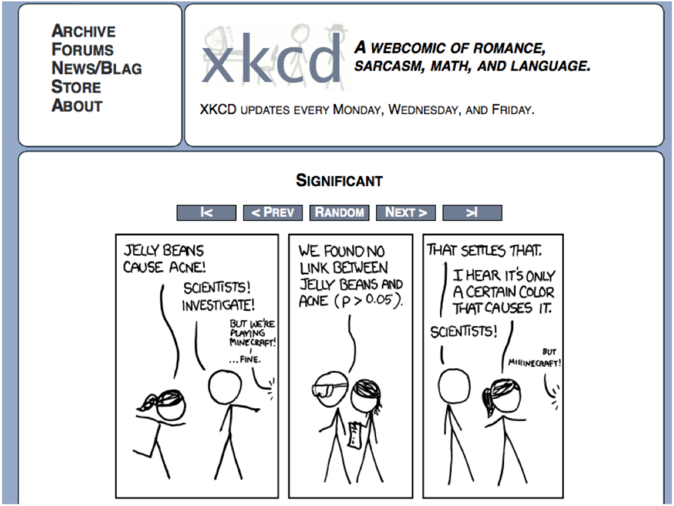
It only works for one test.

Star Chasing (aka there is an XKCD for everything)



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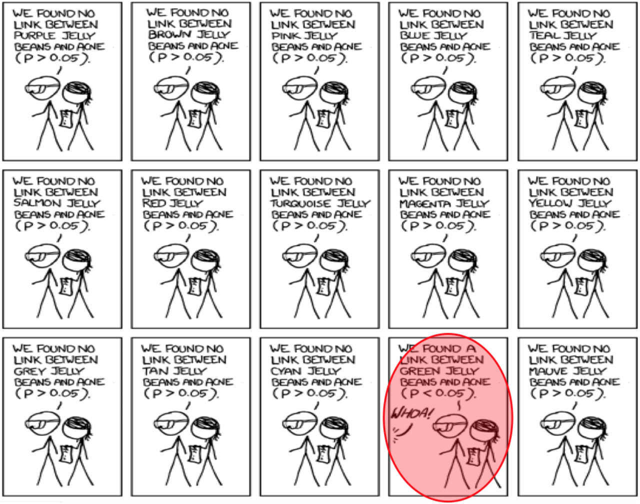

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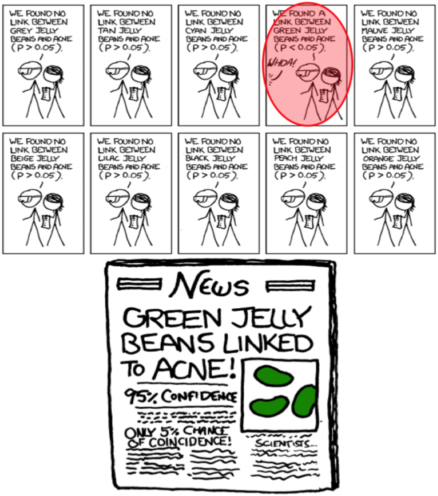

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- Illustration: randomly draw 21 variables, and run a regression of the first variable on the rest.
- By design, no effect of any variable on any other, but when we run the regression:

Multiple Test Example

```
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.0280393  0.1138198  -0.246  0.80605
## X2          -0.1503904  0.1121808  -1.341  0.18389
## X3           0.0791578  0.0950278   0.833  0.40736
## X4          -0.0717419  0.1045788  -0.686  0.49472
## X5           0.1720783  0.1140017   1.509  0.13518
## X6           0.0808522  0.1083414   0.746  0.45772
## X7           0.1029129  0.1141562   0.902  0.37006
## X8          -0.3210531  0.1206727  -2.661  0.00945 **
## X9          -0.0531223  0.1079834  -0.492  0.62412
## X10          0.1801045  0.1264427   1.424  0.15827
## X11          0.1663864  0.1109471   1.500  0.13768
## X12          0.0080111  0.1037663   0.077  0.93866
## X13          0.0002117  0.1037845   0.002  0.99838
## X14         -0.0659690  0.1122145  -0.588  0.55829
## X15         -0.1296539  0.1115753  -1.162  0.24872
## X16         -0.0544456  0.1251395  -0.435  0.66469
## X17          0.0043351  0.1120122   0.039  0.96923
## X18         -0.0807963  0.1098525  -0.735  0.46421
## X19         -0.0858057  0.1185529  -0.724  0.47134
## X20         -0.1860057  0.1045602  -1.779  0.07910 .
## X21          0.0021111  0.1081179   0.020  0.98447
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9992 on 79 degrees of freedom
## Multiple R-squared:  0.2009, Adjusted R-squared:  -0.00142
## F-statistic: 0.993 on 20 and 79 DF,  p-value: 0.4797
```

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- Also note that $2/20 = 0.1$ are significant at the 0.1 level. Totally expected!
- The procedure by which tests/analyses are performed and shown to us matters a lot!

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- Even if all null hypotheses are true we will reject at least one of them with probability .52.
- Same for confidence intervals: probability that all 7 CI cover the true values simultaneously over repeated samples is .52.
  So for each coefficient you have a .90 confidence interval, but overall a .52 percent confidence interval.

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Two Styles of Solutions:
(1) statistical
and
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    - ⋆ Reject all H_k for $k \leq k^*$

Benjamini-Hochberg Example

Pvalues: 0.011, 0.13, 0.06, 0.54, 0.008, 0.024, 0.001, 0.201, 0.78, 0.023

Step 1: Sort	Step 2: Find New Threshold	Step 3: Find k	Step 4: Reject
0.001			
0.008			
0.011			
0.023			
0.024			
0.06			
0.13			
0.201			
0.54			
0.78			

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Step 1: Sort	Step 2: Find New Threshold	Step 3: Find k	Step 4: Reject
0.001	$0.05*1/10 = 0.005$		
0.008	$0.05*2/10 = 0.01$		
0.011	$0.05*3/10 = 0.015$		
0.023	$0.05*4/10 = 0.02$		
0.024	$0.05*5/10 = 0.025$		
0.06	$0.05*6/10 = 0.03$		
0.13	$0.05*7/10 = 0.035$		
0.201	$0.05*8/10 = 0.04$		
0.54	$0.05*9/10 = 0.045$		
0.78	$0.05*10/10 = 0.05$		

Benjamini-Hochberg Example

Pvalues: 0.011, 0.13, 0.06, 0.54, 0.008, 0.024, 0.001, 0.201, 0.78, 0.023

Step 1: Sort	Step 2: Find New Threshold	Step 3: Find k	Step 4: Reject
0.001	0.05*1/10 = 0.005	*	
0.008	0.05*2/10 = 0.01	*	
0.011	0.05*3/10 = 0.015	*	
0.023	0.05*4/10 = 0.02		
0.024	0.05*5/10 = 0.025	*	
0.06	0.05*6/10 = 0.03		
0.13	0.05*7/10 = 0.035		
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0.008	$0.05*2/10 = 0.01$	*	Reject
0.011	$0.05*3/10 = 0.015$	*	Reject
0.023	$0.05*4/10 = 0.02$		Reject
0.024	$0.05*5/10 = 0.025$	*	Reject
0.06	$0.05*6/10 = 0.03$		
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Fun With Salmon

Fun With Salmon

Bennett, Baird, Miller and Wolford. (2009). "Neural correlates of interspecies perspective taking in the post-mortem Atlantic Salmon: an argument for multiple comparisons correction."

Methods

Methods

(a.k.a. the greatest methods section of all time)

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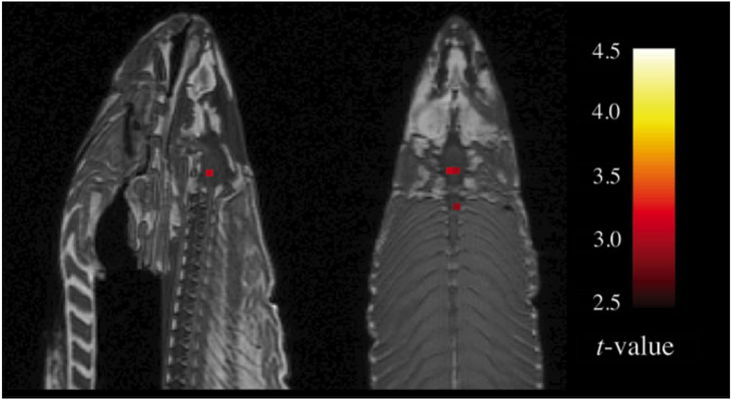
- Design
  "Stimuli were presented in a block design with each photo presented for 10 seconds followed by 12 seconds of rest.A total of 15 photos were displayed. Total scan time was 5.5 minutes."

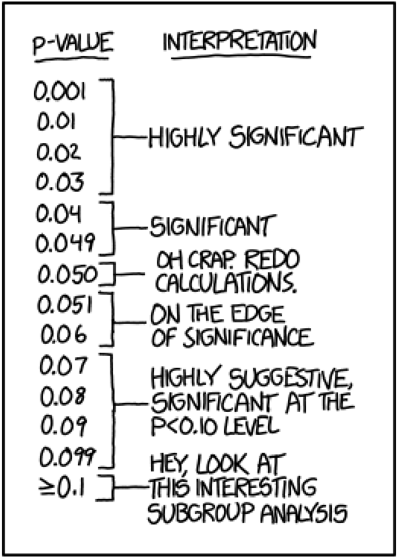
Results



"Several active voxels were discovered in a cluster located within the salmon's brain cavity. The size of this cluster was 81 mm^3 with a cluster-level significance of $p = .001$."

Okay, but what do they mean?

The Meaning of *p*-values (courtesy of XKCD)



The value of the *p*-value

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Ronald Fisher (1935)

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In social science (and I think in psychology as well), the null hypothesis is almost certainly false, false, false, and you don't need a p-value to tell you this. The p-value tells you the extent to which a certain aspect of your data are consistent with the null hypothesis. A lack of rejection doesn't tell you that the null hypothesis is likely true; rather, it tells you that you don't have enough data to reject the null hypothesis.

Andrew Gelman (2010)

Practical versus Statistical Significance

$$T_n = \frac{\hat{\theta} - \theta_0}{\widehat{\text{SE}}[\hat{\theta}]} = \frac{\bar{X} - \mu_0}{\sqrt{V[X]/n}}$$

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- In large samples even tiny effects will be significant, but the results may not be very important substantively. Always discuss both!

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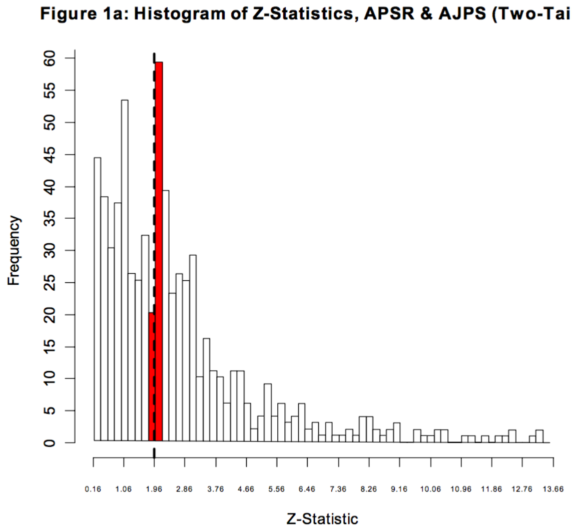
See also: The ASA's (American Statistical Association) Statement on p-Values: Context, Process, and Purpose (`http://dx.doi.org/10.1080/00031305.2016.1154108`)

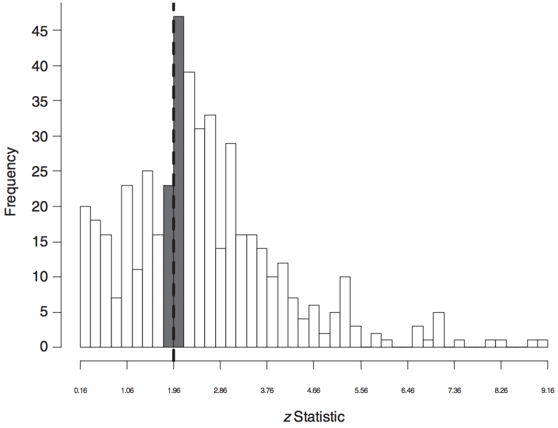
Arbitrary Publication Cutoffs



Gerber and Malhotra (2006) Top Political Science Journals

Arbitrary Publication Cutoffs

Figure 1
Histogram of *z* Statistics From the *American Sociological Review*, the *American Journal of Sociology*, and *The Sociological Quarterly* (Two-Tailed)



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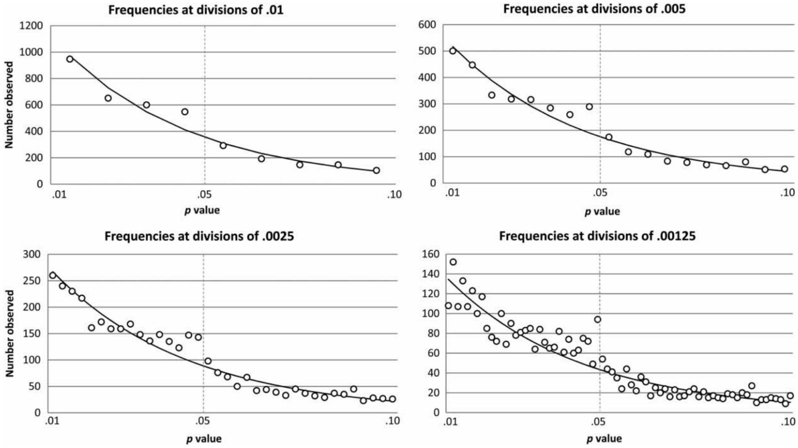

Figure 1.. *The graphs show the distribution of 3,627* p *values from three major psychology journals.*

Masicampo and Lalande (2012) Top Psychology Journals

Still Not Convinced?
The Real Harm of Misinterpreted p-values

ELSEVIER

Accident Analysis and Prevention 36 (2004) 495–500

ACCIDENT ANALYSIS & PREVENTION

www.elsevier.com/locate/aap

Viewpoint

The harm done by tests of significance

Ezra Hauer*

35 Merton Street, Apt. 1706, Toronto, Ont., Canada M4S 3G4

Abstract

Three historical episodes in which the application of null hypothesis significance testing (NHST) led to the mis-interpretation of data are described. It is argued that the pervasive use of this statistical ritual impedes the accumulation of knowledge and is unfit for use.

Keywords: Significance; Statistical hypothesis; Scientific method

Example from Hauer: Right-Turn-On-Red

Table 1
The Virginia RTOR study

	Before RTOR signing	After RTOR signing
Fatal crashes	0	0
Personal injury crashes	43	60
Persons injured	69	72
Property damage crashes	265	277
Property damage (US$)	161243	170807
Total crashes	308	337

The Point in Hauer

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- Two other interesting examples in Hauer (2004)

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The Point in Hauer

- Two other interesting examples in Hauer (2004)
- Core issue is that lack of significance is not an indication of a zero effect, it could also be a lack of power (i.e. a small sample size relative to the difficulty of detecting the effect)
- On the opposite end, large tech companies rarely use significance testing because they have huge samples which essentially always find some non-zero effect. But that doesn't make the finding significant in a colloquial sense of important.

What if I need to show evidence of a zero effect?

AMERICAN JOURNAL of POLITICAL SCIENCE

An Equivalence Approach to Balance and Placebo Tests

Erin Hartman University of California Los Angeles
F. Daniel Hidalgo Massachusetts Institute of Technology

Abstract: *Recent emphasis on credible causal designs has led to the expectation that scholars justify their research designs by testing the plausibility of their causal identification assumptions, often through balance and placebo tests. Yet current practice is to use statistical tests with an inappropriate null hypothesis of no difference, which can result in equating nonsignificant differences with significant homogeneity. Instead, we argue that researchers should begin with the initial hypothesis that the data are inconsistent with a valid research design, and provide sufficient statistical evidence in favor of a valid design. When tests are correctly specified so that difference is the null and equivalence is the alternative, the problems afflicting traditional tests are alleviated. We argue that equivalence tests are better able to incorporate substantive considerations about what constitutes good balance on covariates and placebo outcomes than traditional tests. We demonstrate these advantages with applications to natural experiments.*

Replication Materials: The data, code, and any additional materials required to replicate all analyses in this article are available on the *American Journal of Political Science* Dataverse within the Harvard Dataverse Network, at: https://doi.org/10.7910/DVN/RYNSDG.

Equivalence Tests

Solution: Flip the hypotheses:

$$H_0 : \theta_T - \theta_C \leq \epsilon_L \text{ or } \theta_T - \theta_C \geq \epsilon_U$$
versus
$$H_A : \epsilon_L < \theta_T - \theta_C < \epsilon_U$$

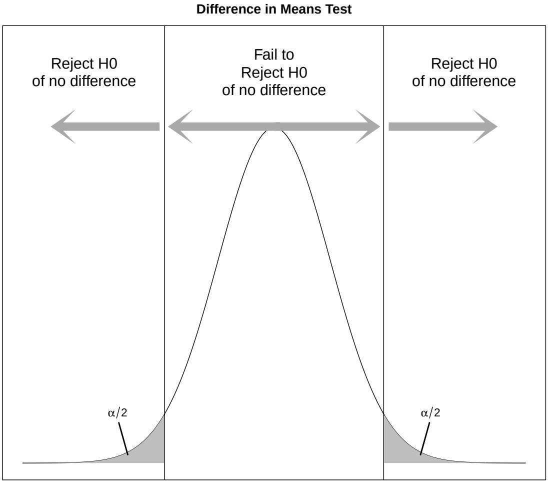
Tests of Difference vs. Equivalence Tests

$$H_0 : \frac{\mu_T - \mu_C}{\sigma} = 0 \qquad \text{versus} \qquad H_A : \frac{\mu_T - \mu_C}{\sigma} \neq 0$$

Type I Error

Test has α probability of declaring the two means different when they are, in fact, the same.

Problem: Controlling for the incorrect type of error if we're trying to provide evidence in favor of equivalence.



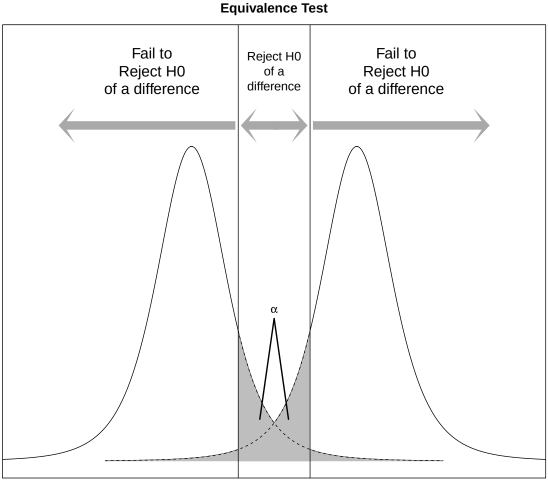
Tests of Difference vs. **Equivalence Tests**

$H_0 : \frac{\mu_T - \mu_C}{\sigma} \geq \epsilon_U$ or $\frac{\mu_T - \mu_C}{\sigma} \leq \epsilon_L$ versus $H_A : \epsilon_L < \frac{\mu_T - \mu_C}{\sigma} < \epsilon_U$

Type I Error

Test has α probability of declaring the two means equivalent when they are, in fact, the different.

Solution: Now control for the correct type of false positive.



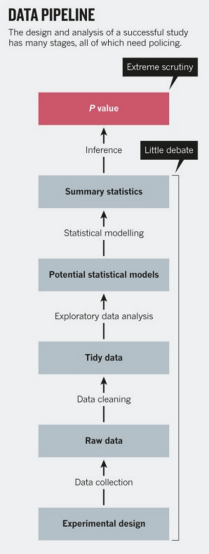
p-values

p-values

General Message:

Don't misinterpret, or rely too heavily, on your p-values. They are evidence against your null, not evidence in favor of your alternative.

But Let's Not Obsess Too Much About p-values



From Leek and Peng (2015) "*P* values are just the tip of the iceberg" *Nature*.

We Covered

We Covered

- p-values

We Covered

- p-values
- multiple testing

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Next Time: What is Regression?

We Covered

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Next Time: What is Regression?

Bonus reading for those interested to learn more:

- Hauer. 2004 "The harm done by tests of significance." *Accident Analysis & Prevention*.
- Gigerenzer. 2004. "Mindless statistics." *Journal of Socio-Economics*.
- Nuzzo. 2014. "Statistical Errors." *Nature*
- Ward et al. 2010. "The perils of policy by p-value: Predicting civil conflicts." *Journal of Peace Research*
- Cohen. 1994. "The Earth is Round ($p < 0.05$)." *American Psychologist*
- Schwab. 2011. "Researchers should make thoughtful assessments instead of null-hypothesis significance tests." *Organizational Science*.

Where We've Been and Where We're Going...

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- Last Week
  - inference and estimator properties
  - point estimates, confidence intervals
- This Week
  - hypothesis testing
  - what is regression?
  - nonparametric and linear regression
- Next Week
  - inference for simple regression
  - properties of ordinary least squares
- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causal inference

1. Hypothesis Testing
   - Terminology and Procedure
   - One-Sided Tests
   - Connections
   - Power
2. p-values
   - Mechanics
   - Multiple Testing
   - Fun With Salmon
   - The Significance of Significance
3. What is Regression?
   - Conditional Expectation Functions
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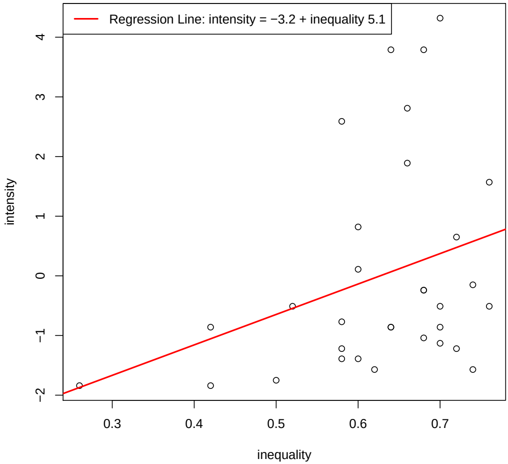
What You've Probably Seen This



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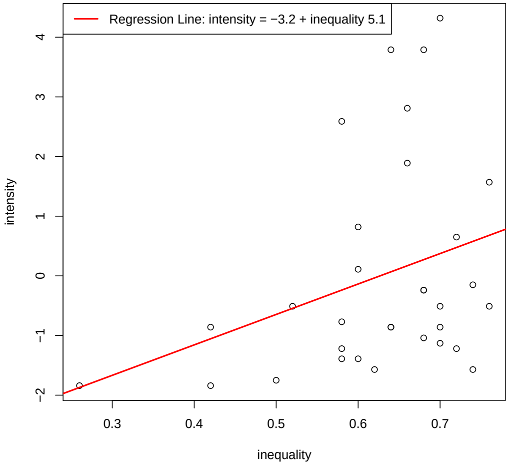

We are going to go about this a slightly different way.

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- By convention variables tend to be called X and Y.
- Y - the dependent variable or outcome or regressand or left-hand-side variable or response
  - Voter turnout
  - Receiving a job
  - Income relative to parent

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  - Does turnout vary by types of mailers received?
  - Does a the probability of getting a job vary by applicant race?
  - Is intergenerational mobility less likely for children with an incarcerated parent?
- By convention variables tend to be called X and Y.
- Y - the dependent variable or outcome or regressand or left-hand-side variable or response
  - Voter turnout
  - Receiving a job
  - Income relative to parent
- X - the independent variable or explanatory variable or regressor or right-hand-side variable or treatment or predictor
  - Social pressure mailer versus Civic Duty Mailer
  - Applicant race
  - Incarcerated parent

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  - Causal Inference: with additional assumptions (later in the semester) we can talk about how intervening to change the value of X will change Y.

Reminder of Definitions

Definition (Conditional Expectation (Discrete))

Let Y and X be discrete random variables. The conditional expectation of Y given $X = x$ is defined as:

$$E[Y|X = x] = \sum_y y\, P(Y = y|X = x) = \sum_y y\, p_{Y|X}(y|x)$$

Definition (Conditional Expectation (Continuous))

Let Y and X be continuous random variables. The conditional expectation of Y given $X = x$ is given by:

$$E[Y|X = x] = \int_{-\infty}^{\infty} y\, f_{Y|X}(y|x) dy$$

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- and the CEF is the lowest mean squared error predictor of Y_i given X_i

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$$\mu_1 = E[Y|X = 1]$$
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- Notice here that since X can only take on two values, 0 and 1, then these two conditional means completely summarize the CEF.
- Estimation just involves taking the means within the groups.

Non-binary CEFs

- If X is discrete with not too many categories, we can estimate the conditional expectation using the means within groups:
  - $\hat{\mu}(1) = \hat{E}[Y|X = 1] = \frac{1}{n_{x=1}} \sum_{i:X_i=1} Y_i$
  - $\hat{\mu}(2) = \hat{E}[Y|X = 2] = \frac{1}{n_{x=2}} \sum_{i:X_i=2} Y_i$
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- When X can take on many possible values (think income) or we have few observation for a given value of X, we have to write out a more general function.
- These functional forms are unknown which makes life hard.

1. Hypothesis Testing
   - Terminology and Procedure
   - One-Sided Tests
   - Connections
   - Power
2. p-values
   - Mechanics
   - Multiple Testing
   - Fun With Salmon
   - The Significance of Significance
3. What is Regression?
   - Conditional Expectation Functions
   - Nonparametric Regression
   - Best Linear Predictor
   - Ordinary Least Squares
4. Interpreting Regression
   - Fun With Linearity

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- Let's take a look at some data on education and income from the American National Election Study
- We use two variables:
  - Y: income
  - X: educational attainment
- Goal is to characterize the conditional expectation $E[Y|X = x]$, i.e. how average income varies with education level

Nonparametric Regression with Discrete X

Nonparametric Regression with Discrete X

`educ`: Respondent's education:

- 1. 8 grades or less and no diploma or
- 2. 9-11 grades
- 3. High school diploma or equivalency test
- 4. More than 12 years of schooling, no higher degree
- 5. Junior or community college level degree (AA degrees)
- 6. BA level degrees; 17+ years, no postgraduate degree
- 7. Advanced degree

Nonparametric Regression with Discrete X

`income`: Respondent's family income:

- 1. None or less than \$2,999
- 2. \$3,000-\$4,999
- 3. \$5,000-\$6,999
- 4. \$7,000-\$8,999
- 5. \$9,000-\$9,999
- 6. \$10,000-\$10,999

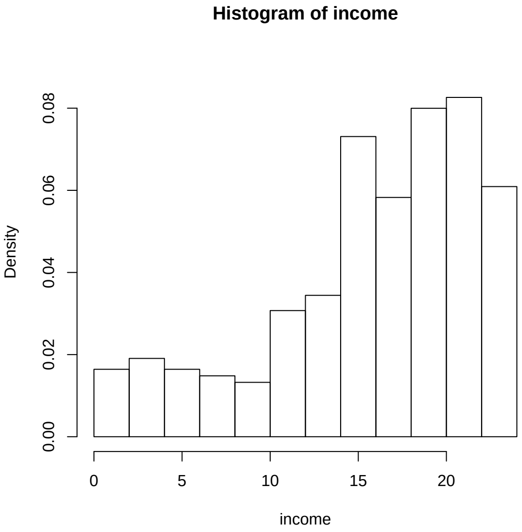
  ⋮

- 17. \$35,000-\$39,999
- 18. \$40,000-\$44,999

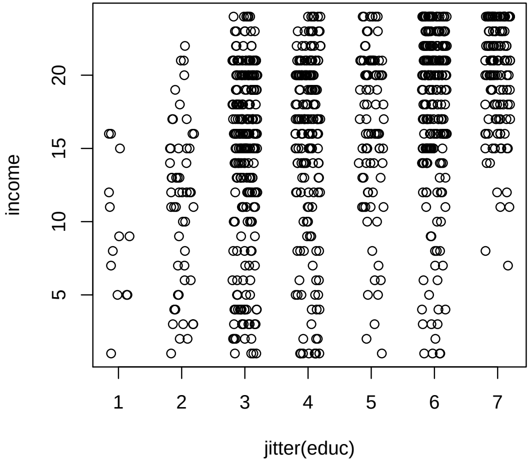
  ⋮

- 23. \$90,000-\$104,999
- 24. \$105,000 and over

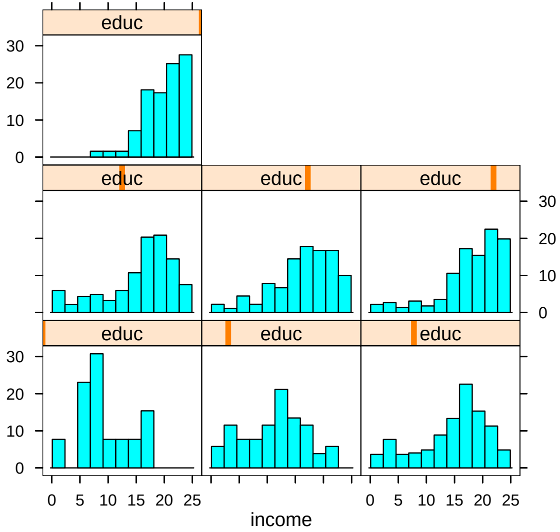
Marginal Distribution of Y (income)



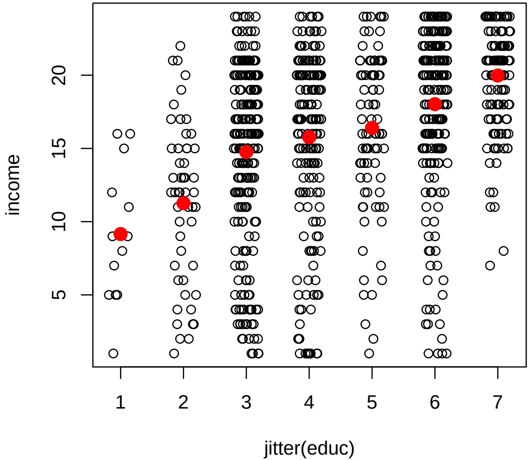
Joint Distribution of X and Y (Income and Education)



Distribution of income given education $p(y|x)$



Nonparametric Regression with Discrete X



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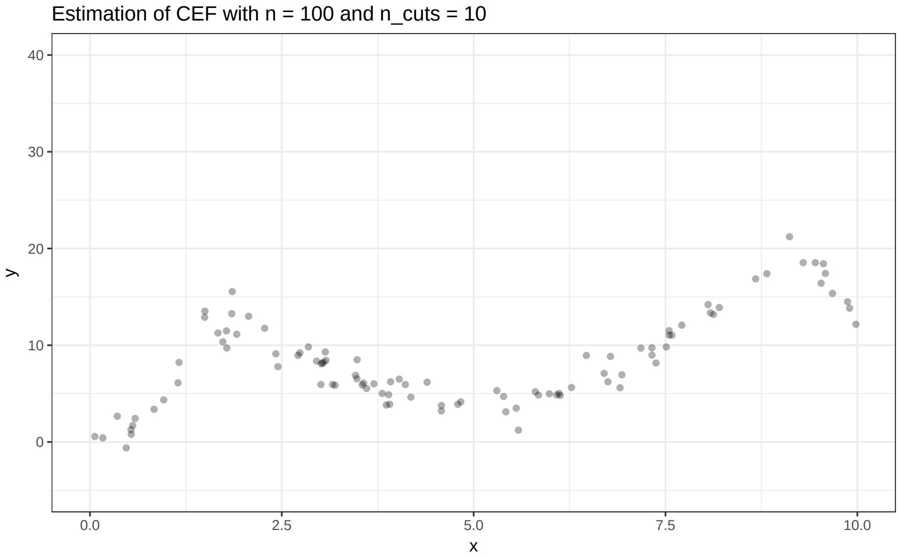
Nonparametric Regression

- This approach works well as long as
  - X is discrete
  - there are a small number values of X
  - a small number of X variables
  - a lot of observations at each X value
- But what do we do when X is continuous and has many values?
- Let's talk through a few options.

A Binning Approach to CEF for continuous random variables



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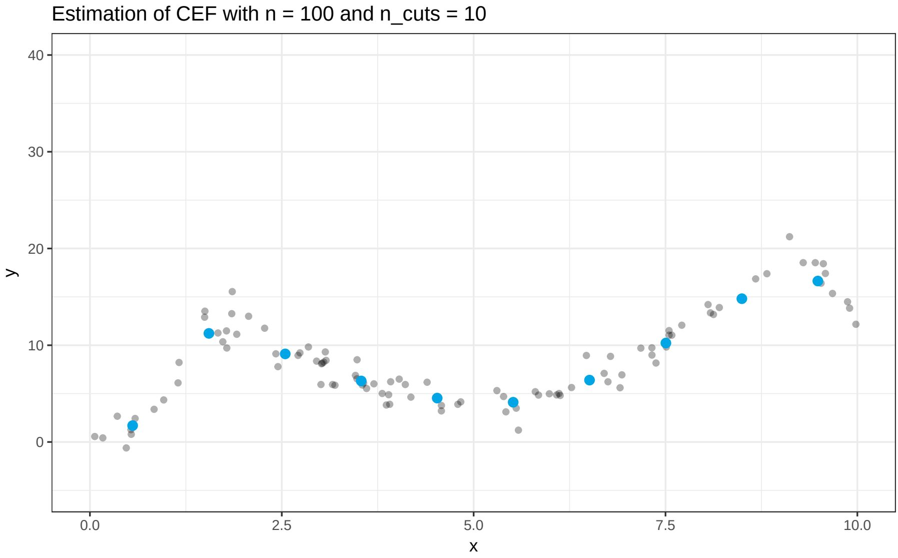

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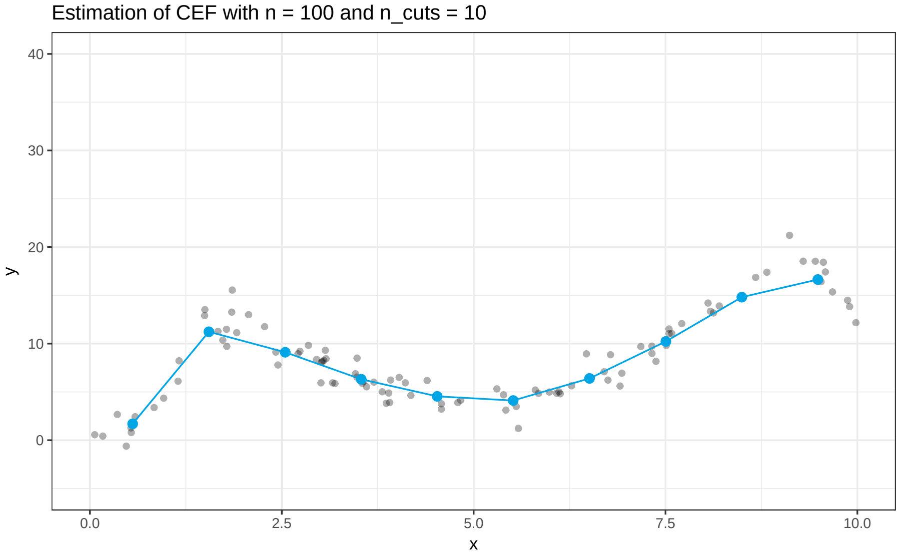

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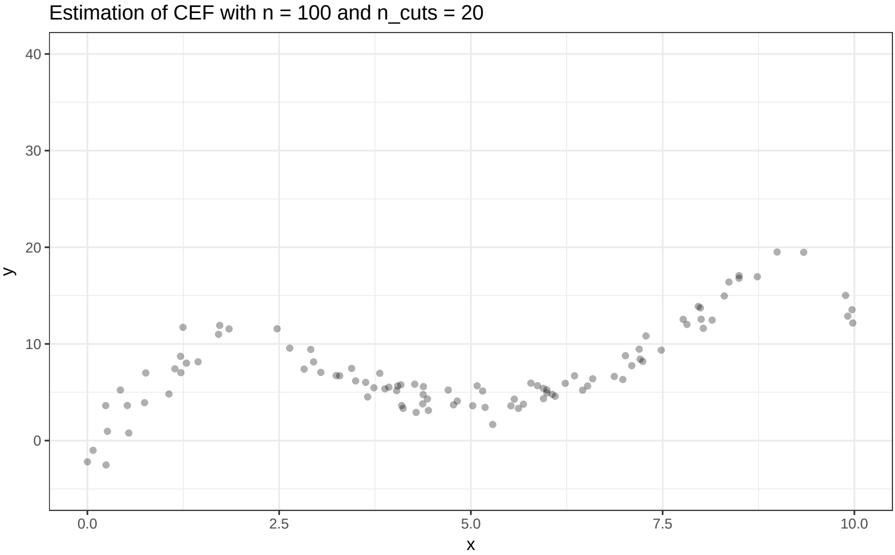

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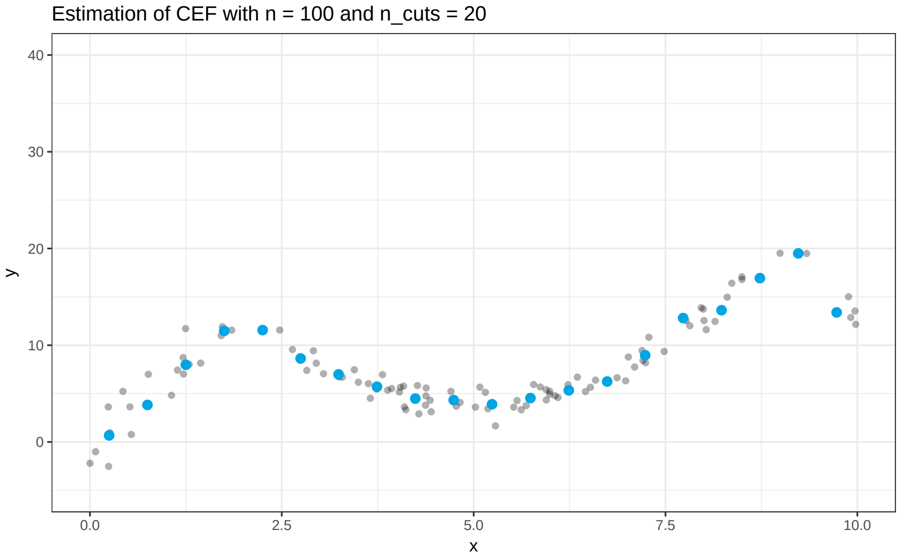

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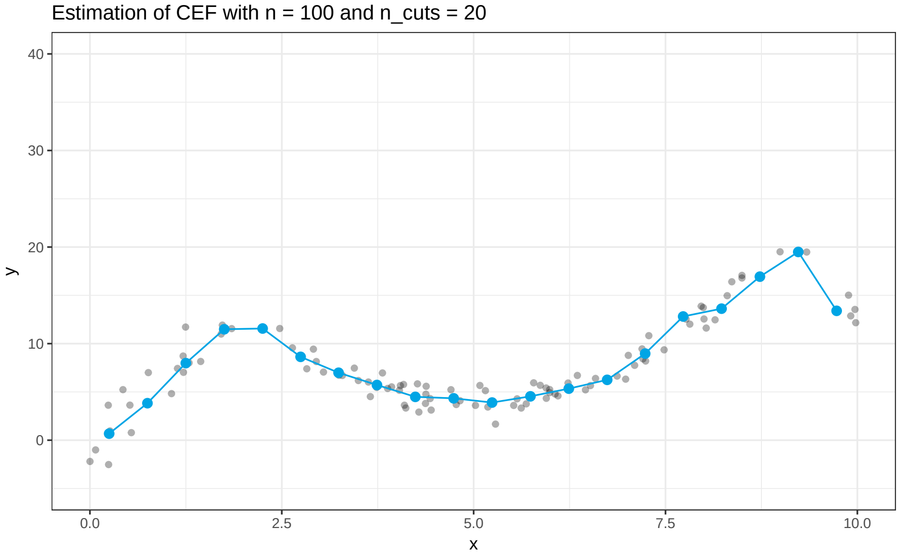
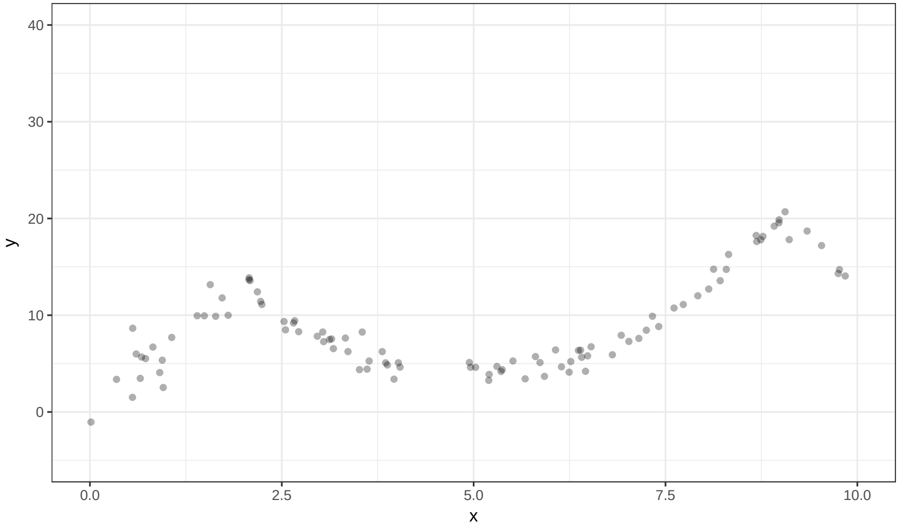


A Binning Approach to CEF for continuous random variables

Estimation of CEF with n = 100 and n_cuts = 50



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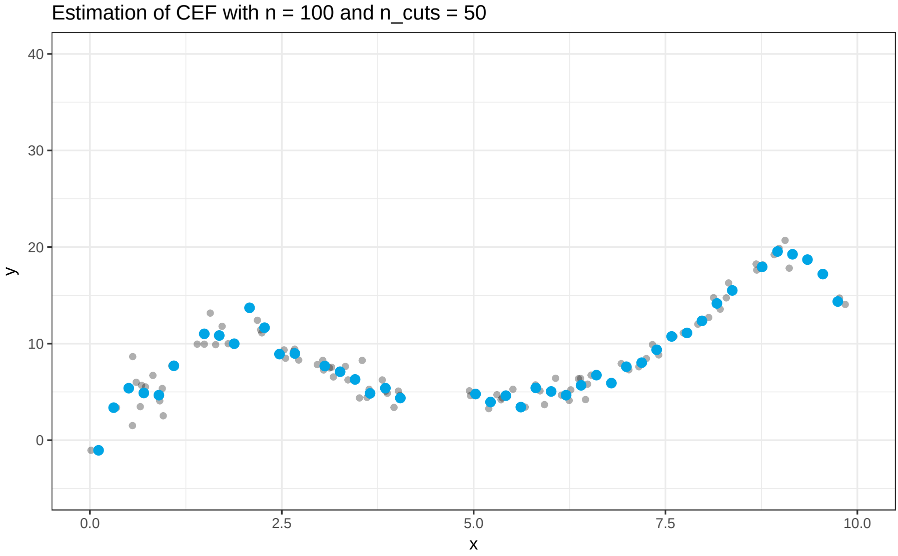

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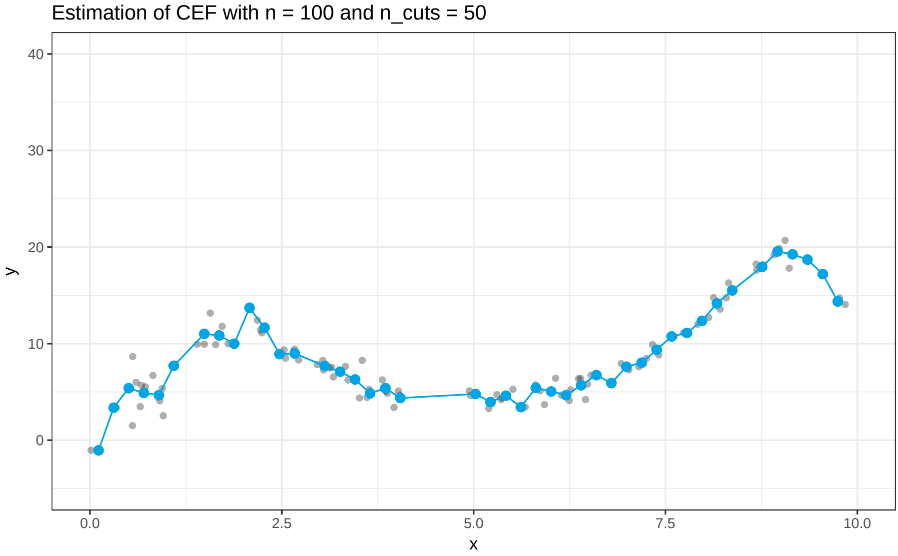

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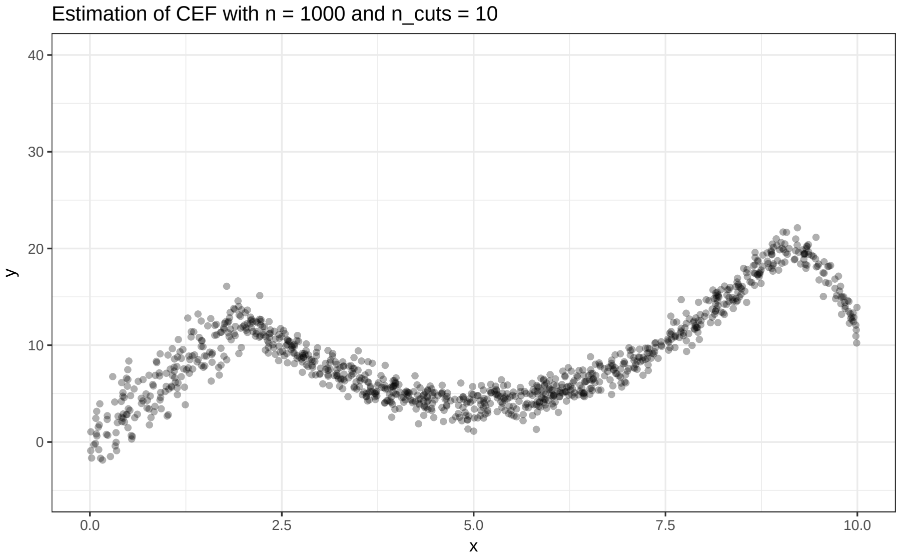

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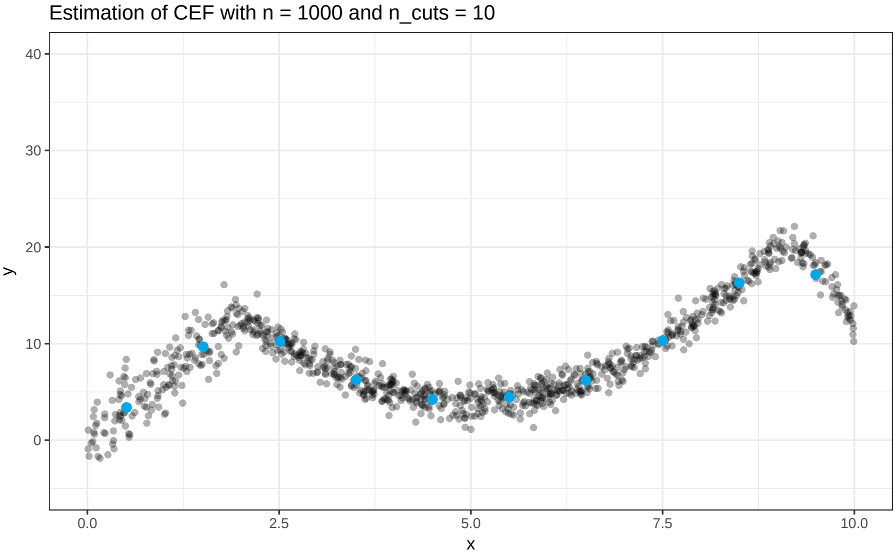

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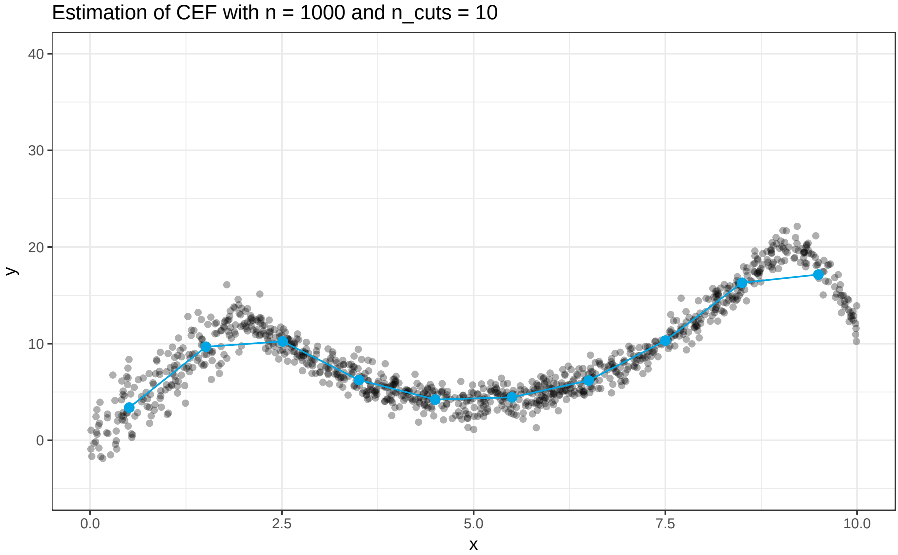

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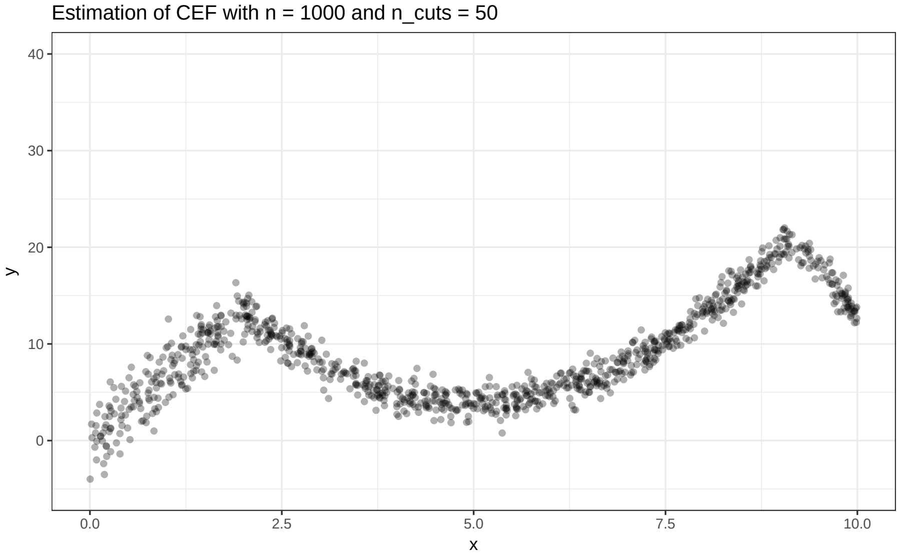

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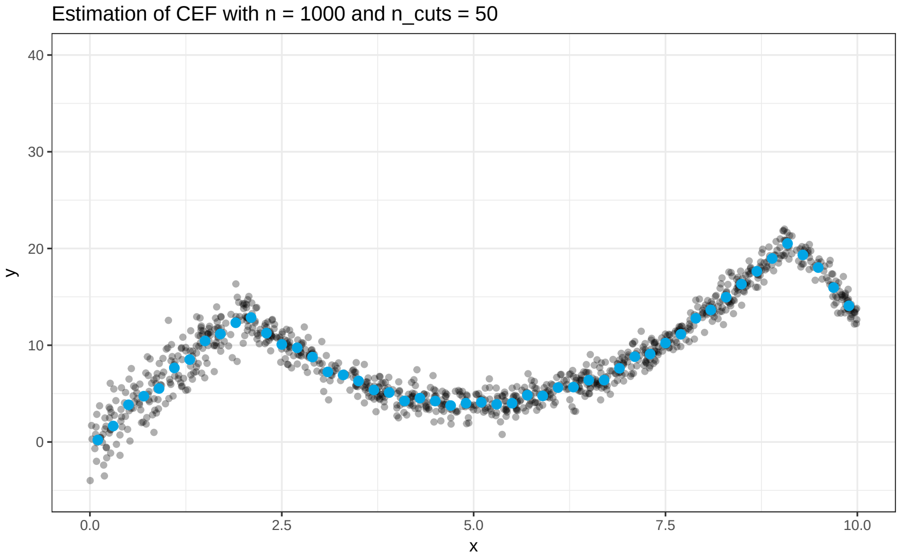

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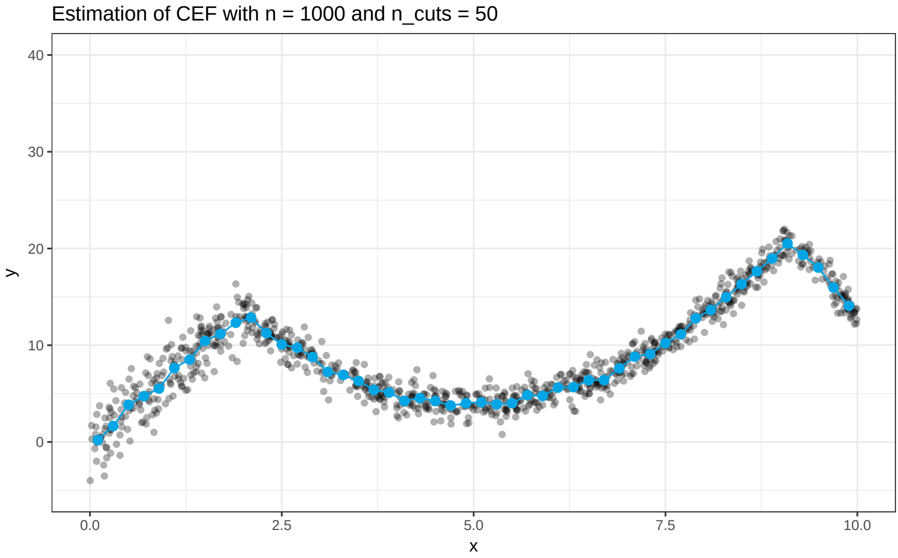

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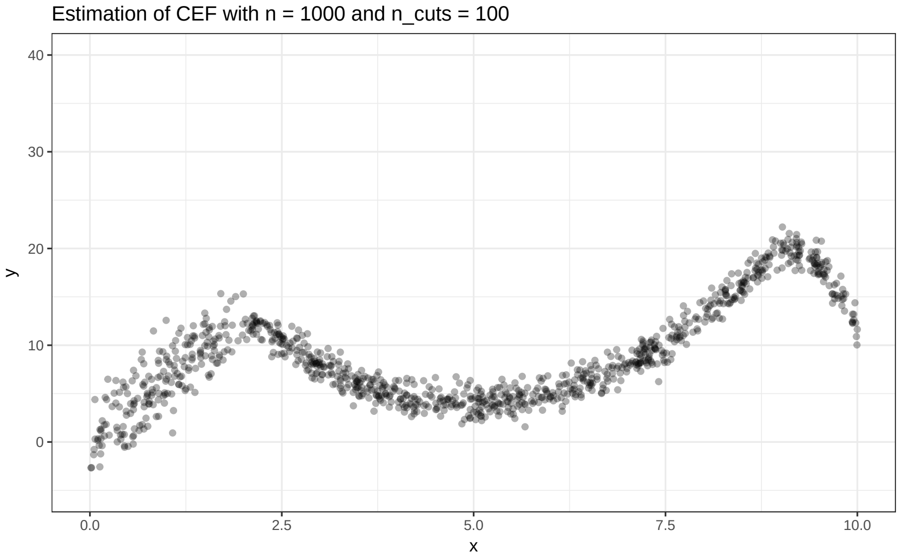

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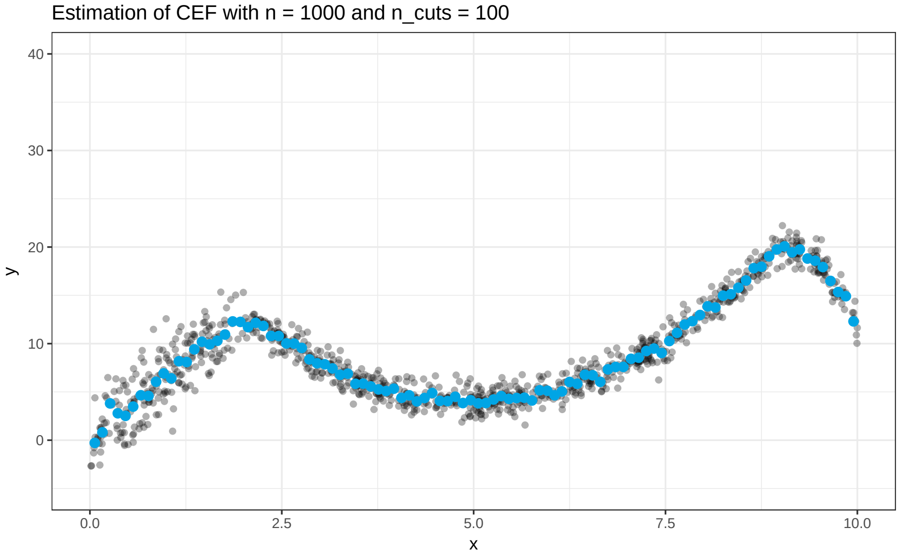

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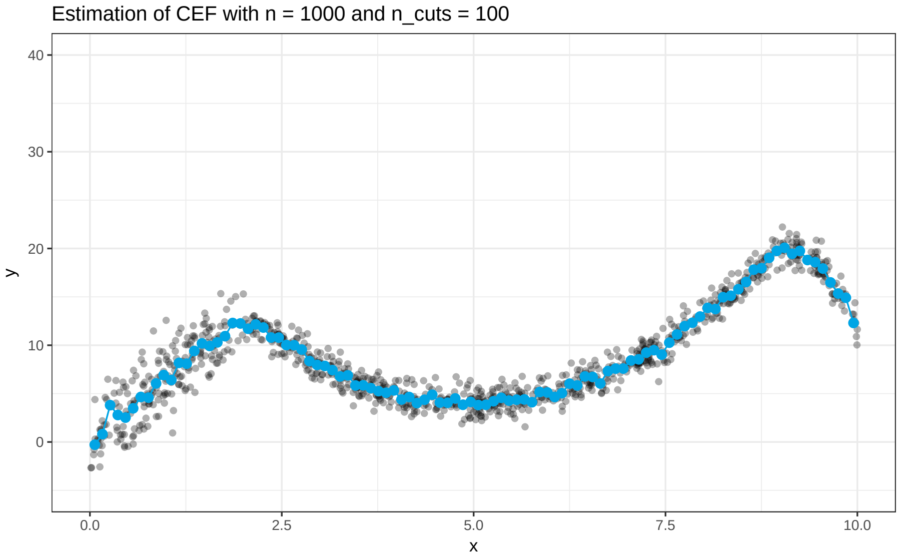

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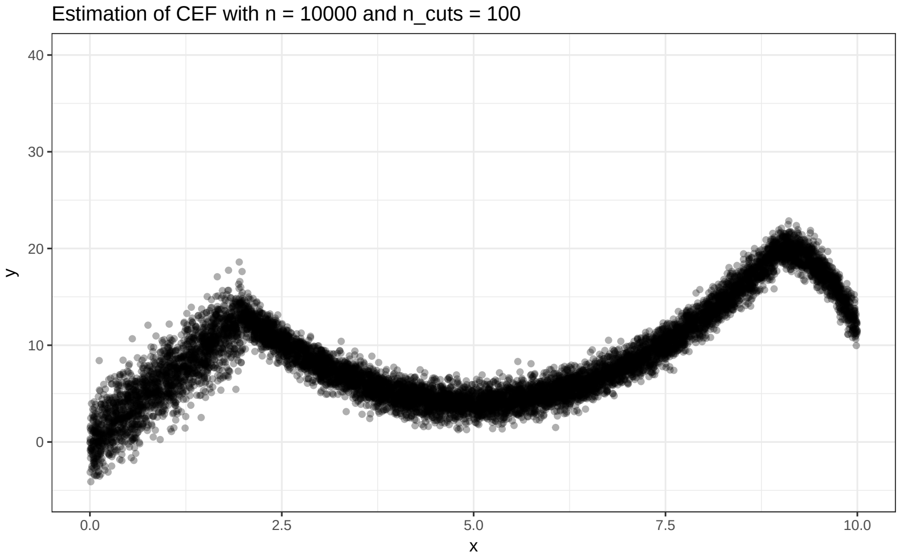

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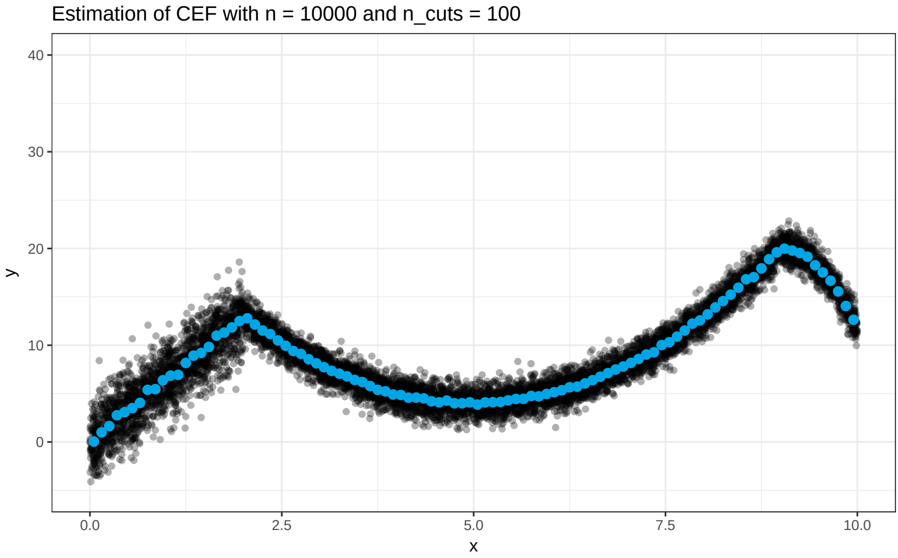

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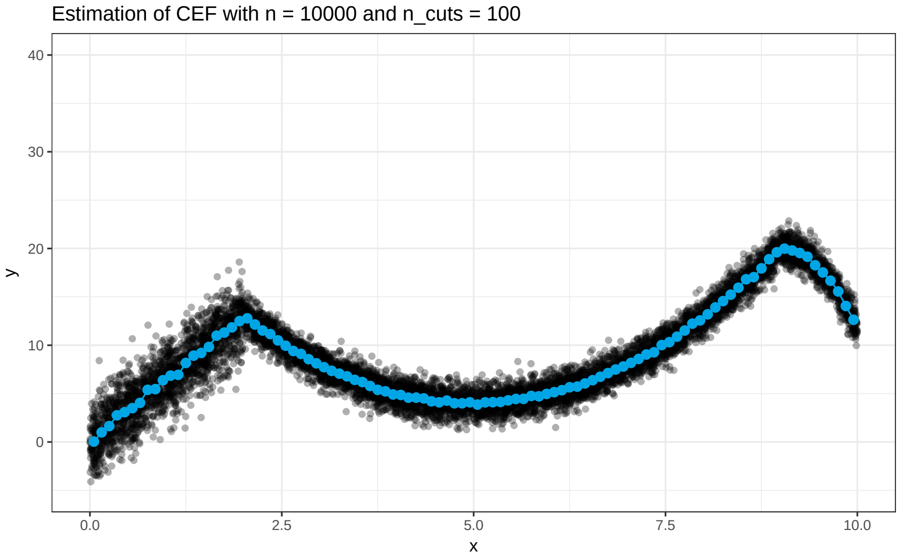

Uniform Kernel Regression: Simple Local Averages

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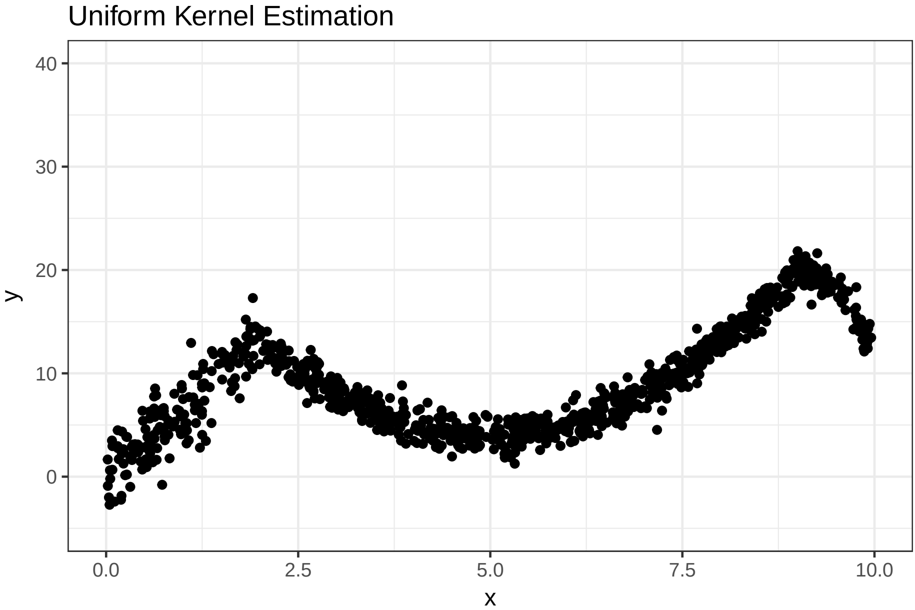
- We will call this approach uniform kernel regression for a reason that will become clear shortly.

Example of Kernel CEF Estimation



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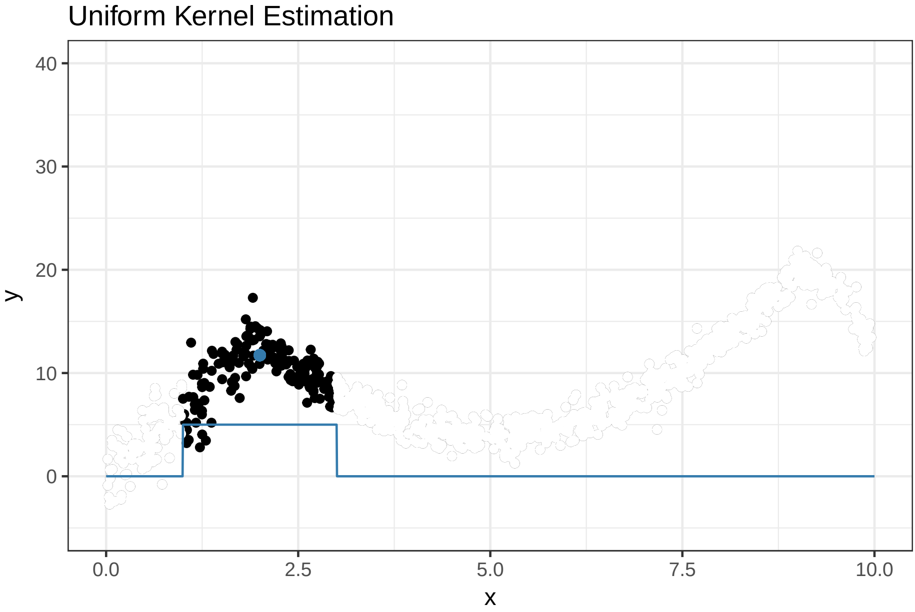

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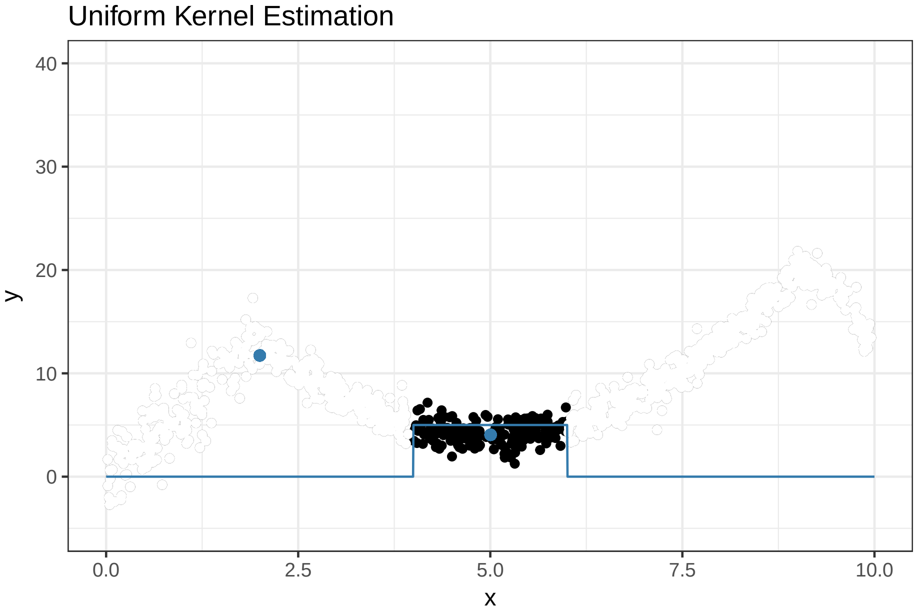

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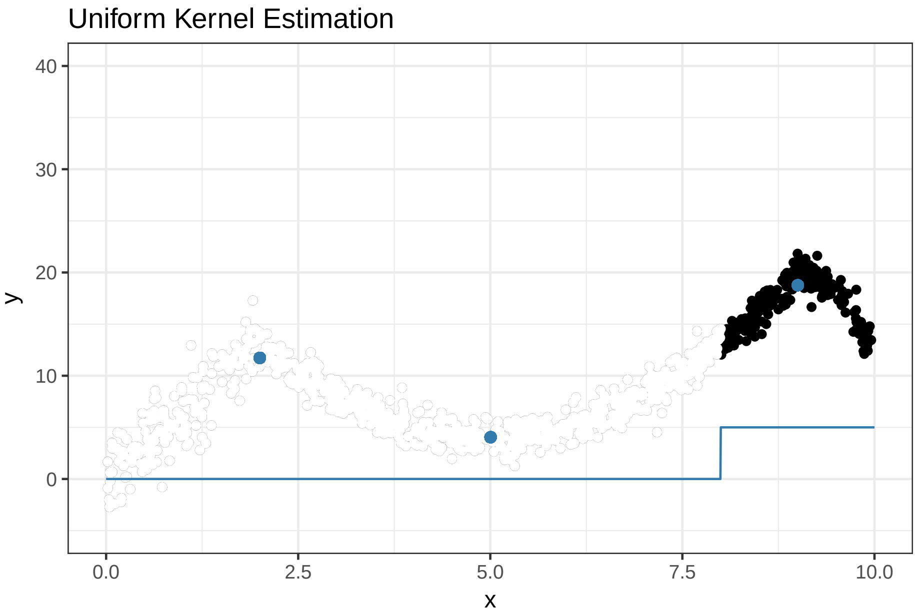

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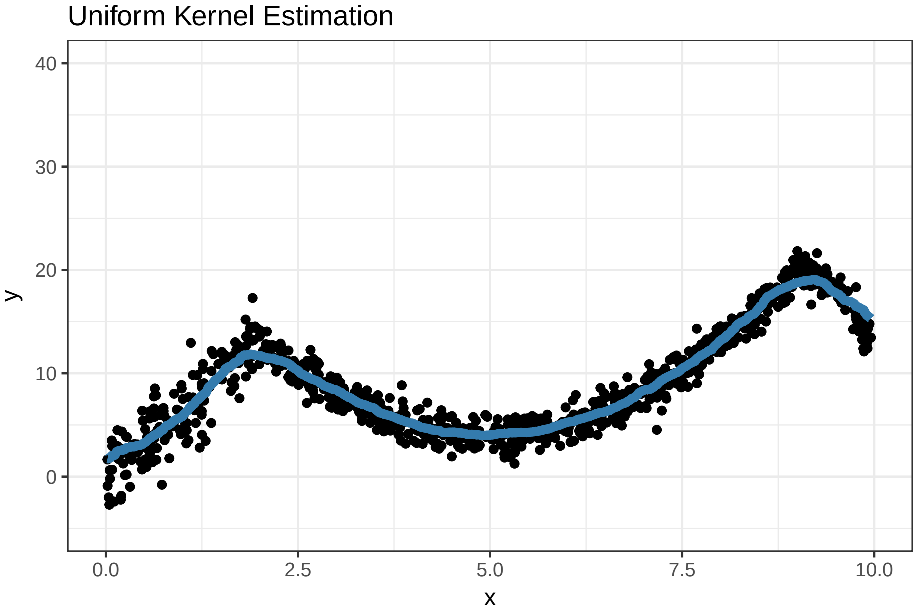

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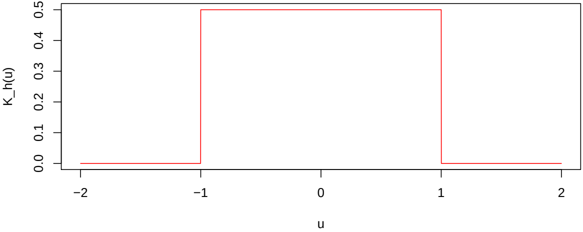

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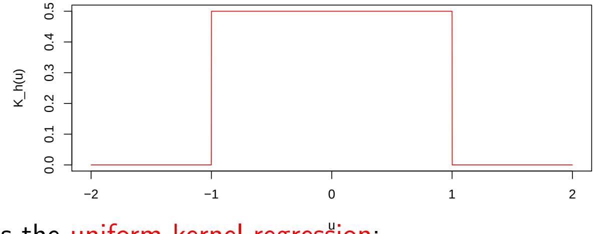
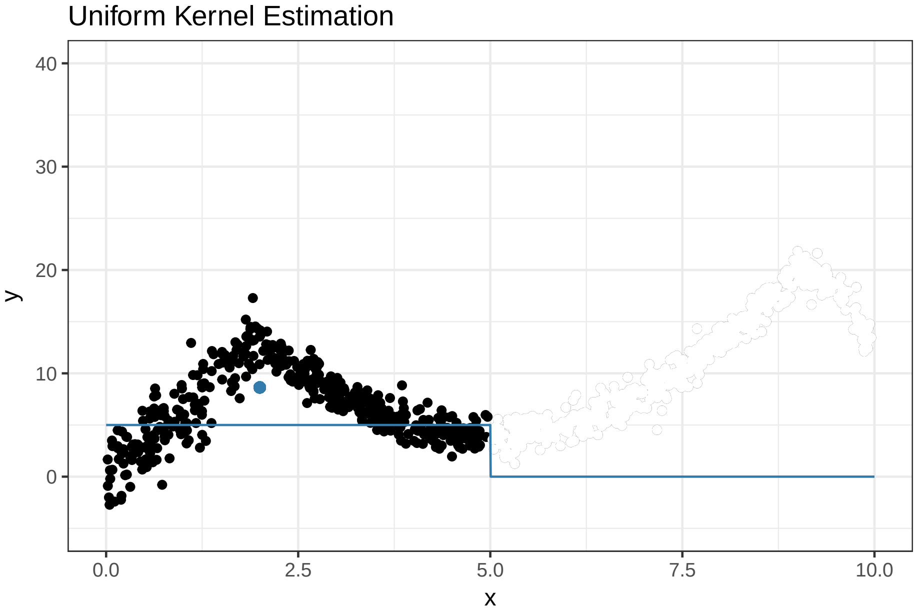


- This gives the uniform kernel regression:

$$\widehat{E}[Y|X = x_0] = \frac{\sum_{i=1}^{N} K_h((X_i - x_0)/h)Y_i}{\sum_{i=1}^{N} K_h((X_i - x_0)/h)} \text{ where } K_h(u) = \frac{1}{2}\,\mathbf{1}_{\{|u|\leq 1\}}$$

Changing the Bandwidth



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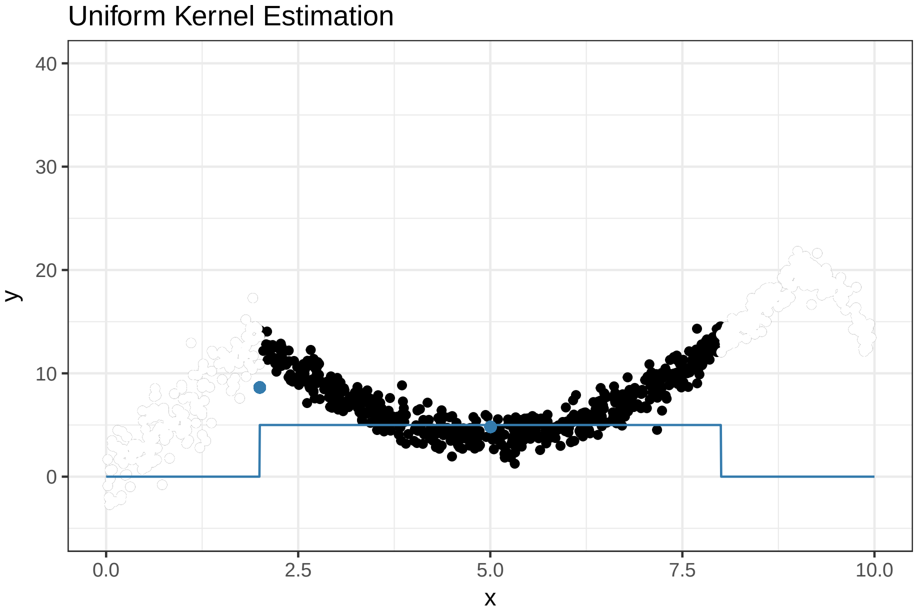

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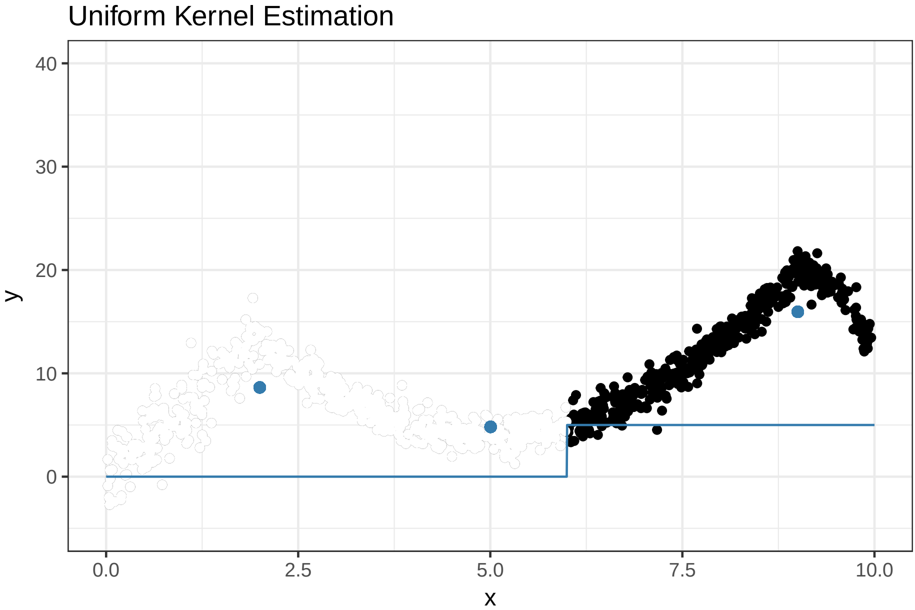

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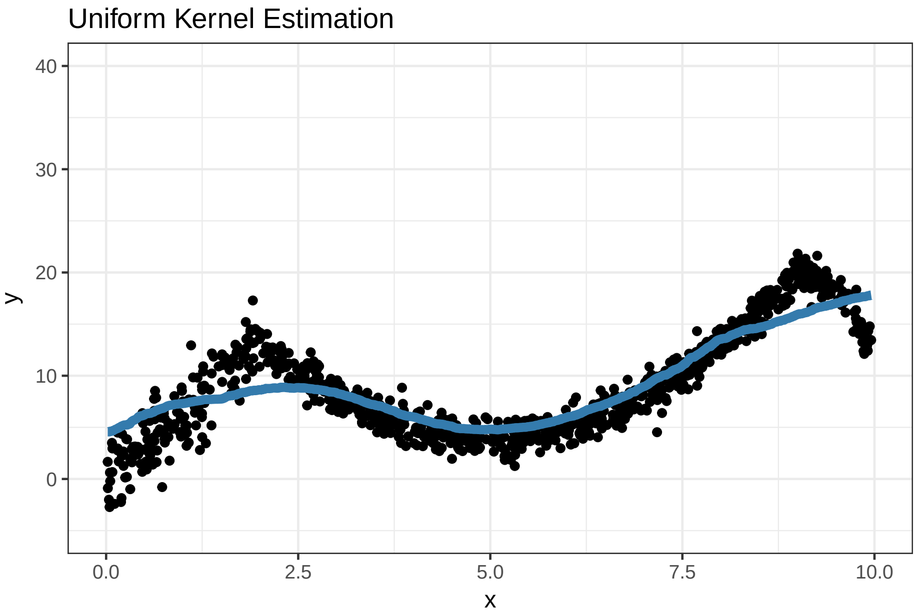

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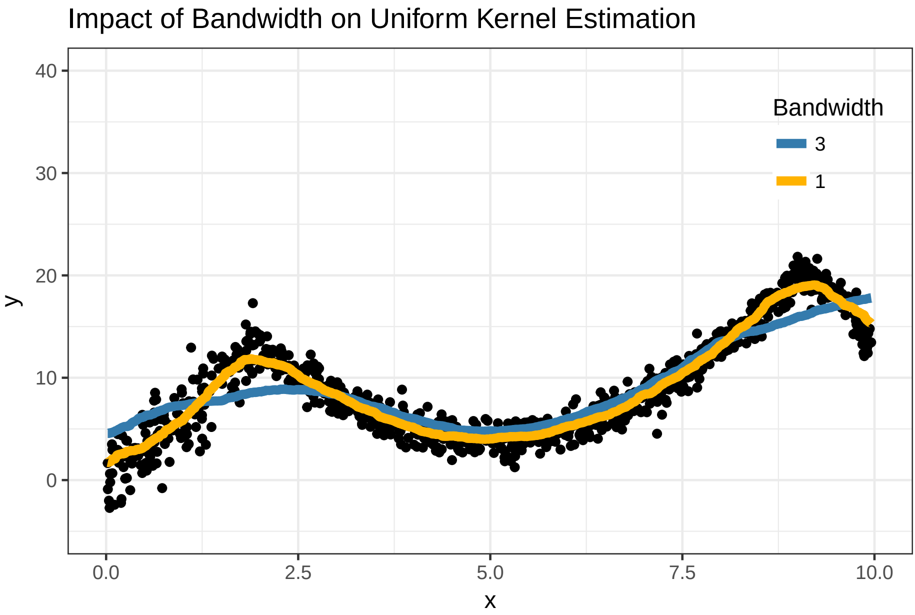

Uniform Kernel Regression: Properties

Theorem (Consistency of the Uniform Kernel Density Estimator)

For iid continuous random variables $X_1, X_2, \ldots X_n$, $\forall x \in \mathbb{R}$,

- *if the kernel is uniform, and*
- *if* $h \to 0$ *and*
- $nh \to \infty$ *as*
- $n \to \infty$, *then*

$$\hat{f}_K(x) \xrightarrow{p} f(x).$$

Aronow and Miller Theorem 3.3.8. Proof by weak law of large numbers and the plug-in principle.

The More General Form of the Estimator

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Definition (Kernel Density Estimator)

Let $X_1, X_2, \dots X_n$ be iid continuous random variables with common PDF f.

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Let $K : \mathbb{R} \to \mathbb{R}$ be a function which is symmetric about the y-axis satisfying $\int_{-\infty}^{\infty} K(x)dx = 1$, and let $K_h(x) = \frac{1}{h}K(\frac{x}{h}) \forall x \in \mathbb{R}$ and $h > 0$.

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Then a kernel density estimator of $f(x)$ is

$$\hat{f}_K(x) = \frac{1}{n}\sum_{i=1}^{n} K_h(x - X_i), \forall x \in \mathbb{R}$$

The function K is called the kernel and the scaling parameter h is called the bandwidth.
(Aronow and Miller Definition 3.3.7)

Kernel Estimation

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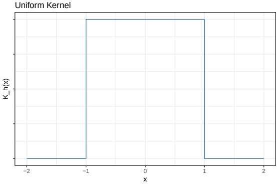
- Uniform Kernel
- Calculate the average of the observed y points that have x values in the interval $[x_0 - h, x_0 + h]$
- Each observation within the interval is given equal weight, each observation outside the interval is given 0 weight



Kernel Estimation

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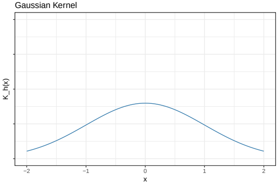
- Gaussian Kernel
- Distance weighted by how far from x_0 following the normal density

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$



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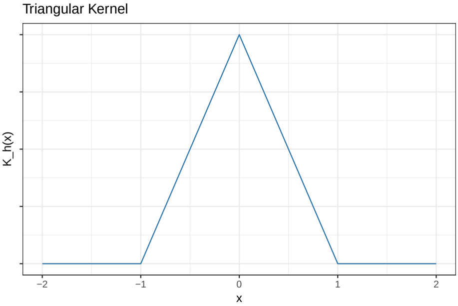
- Distance weighted by how far from x_0 using linear distance



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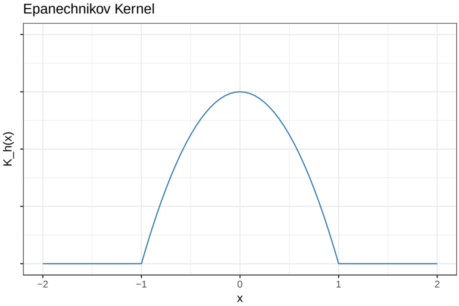
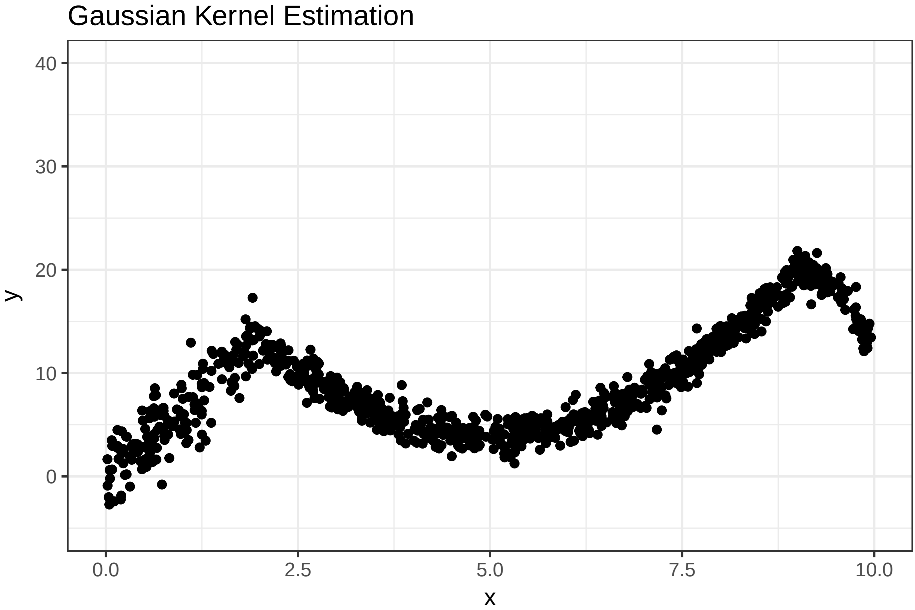


Example of Gaussian Kernel



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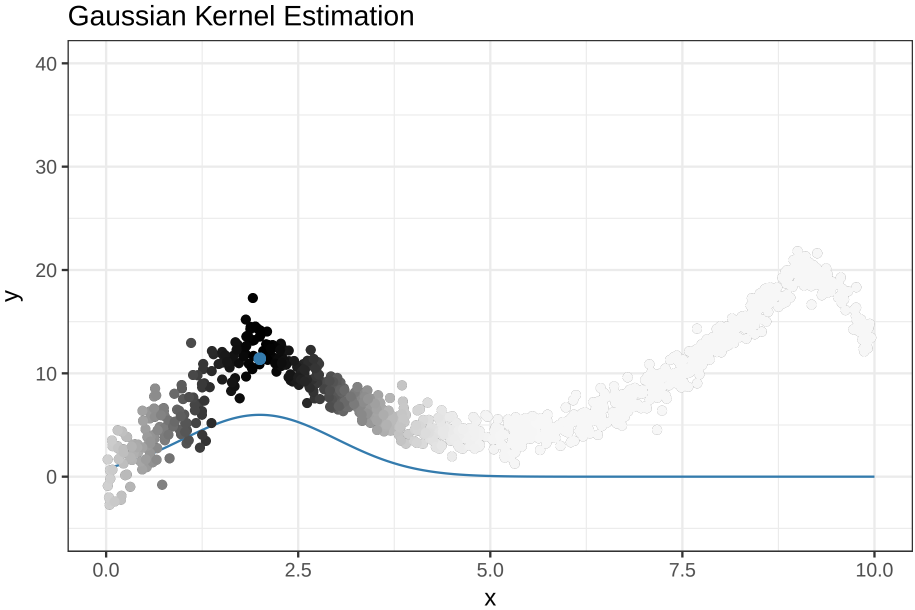

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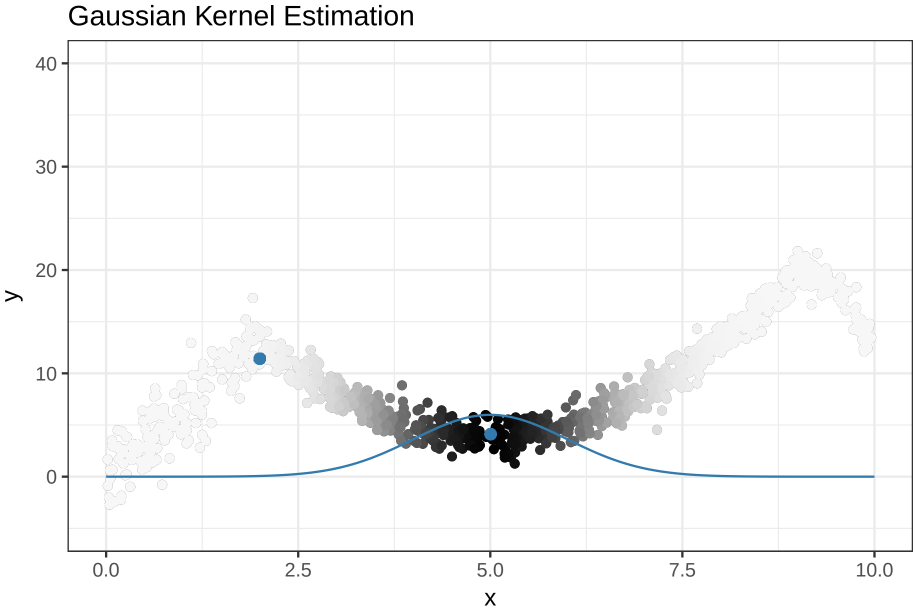

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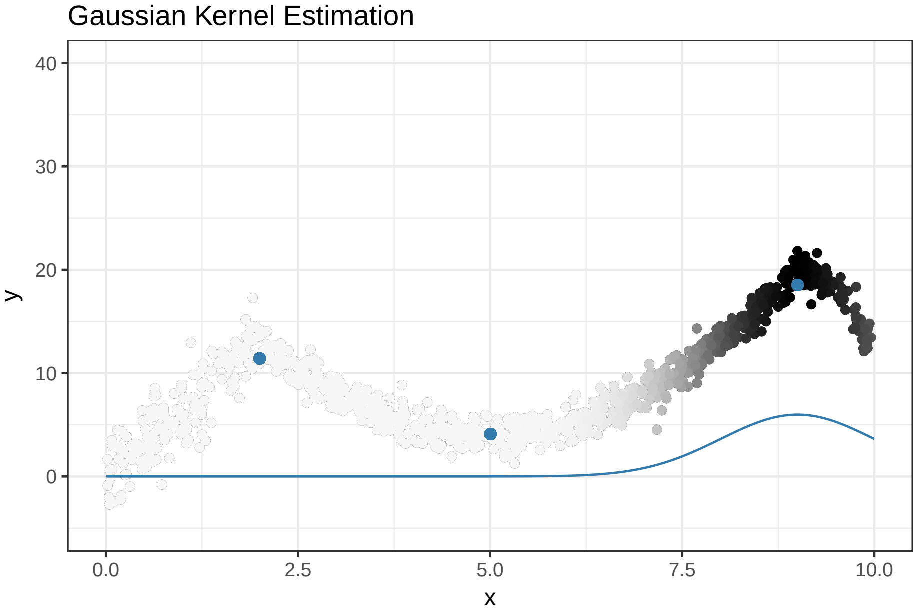

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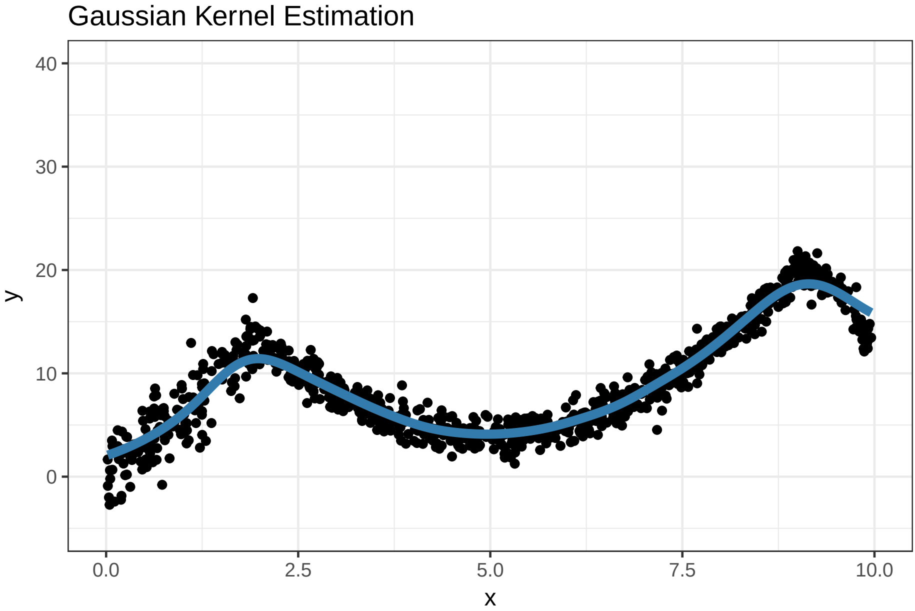

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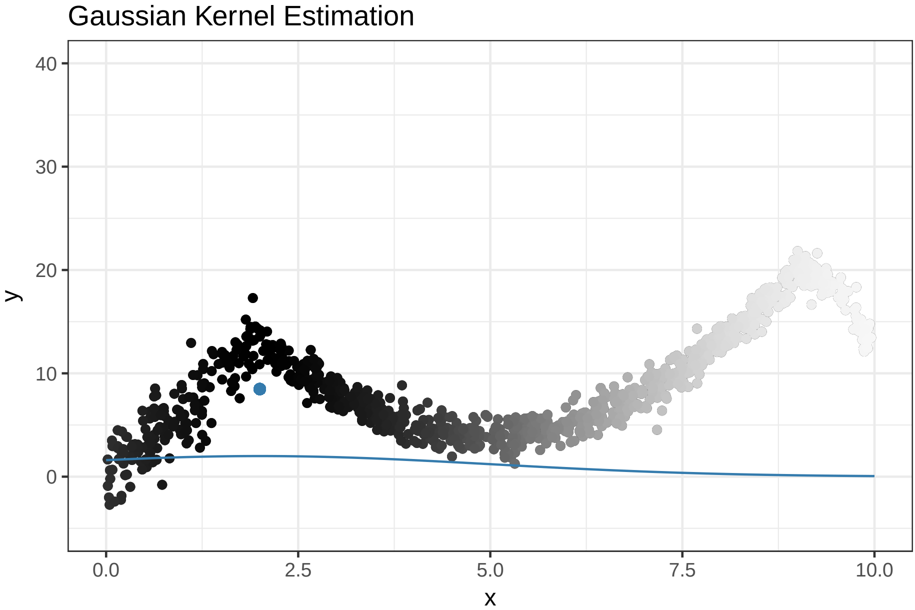

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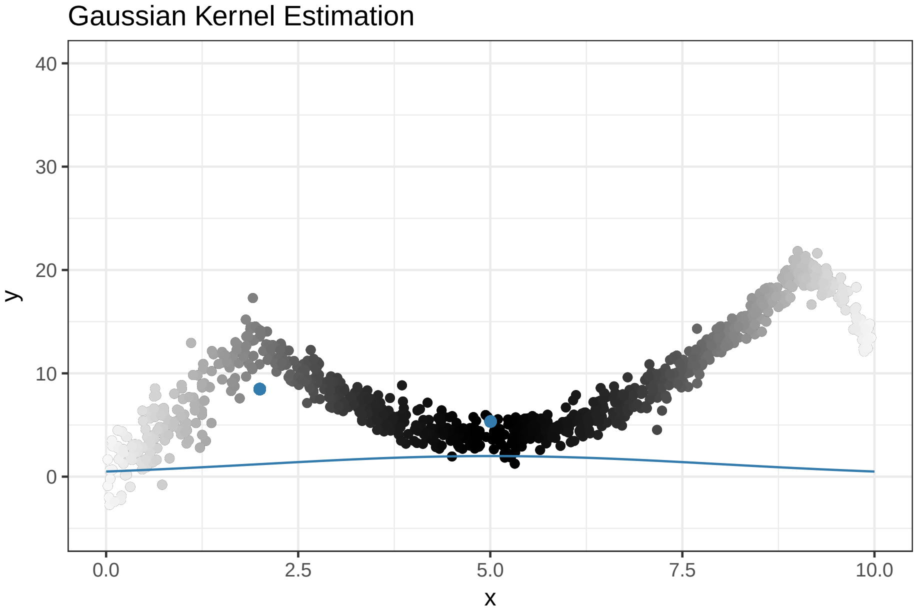

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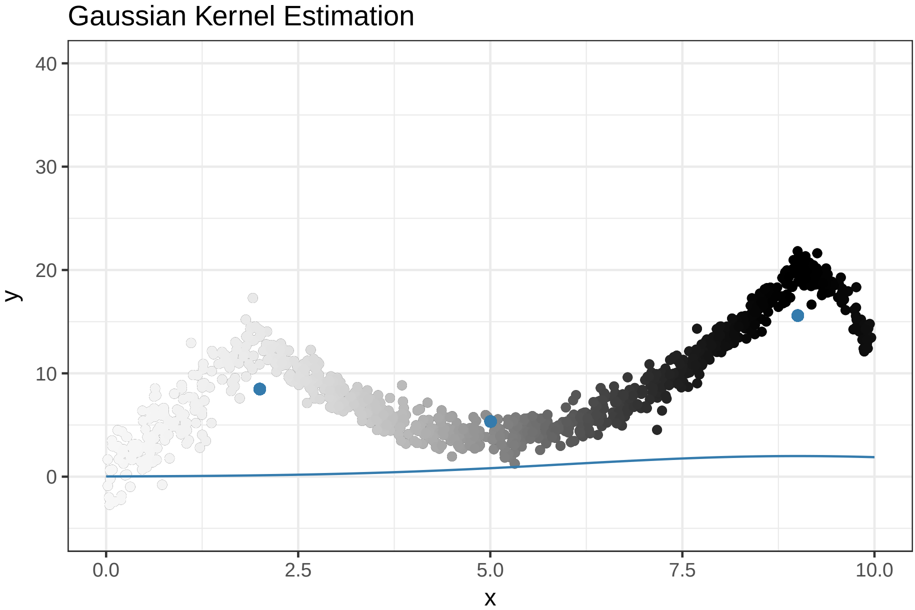

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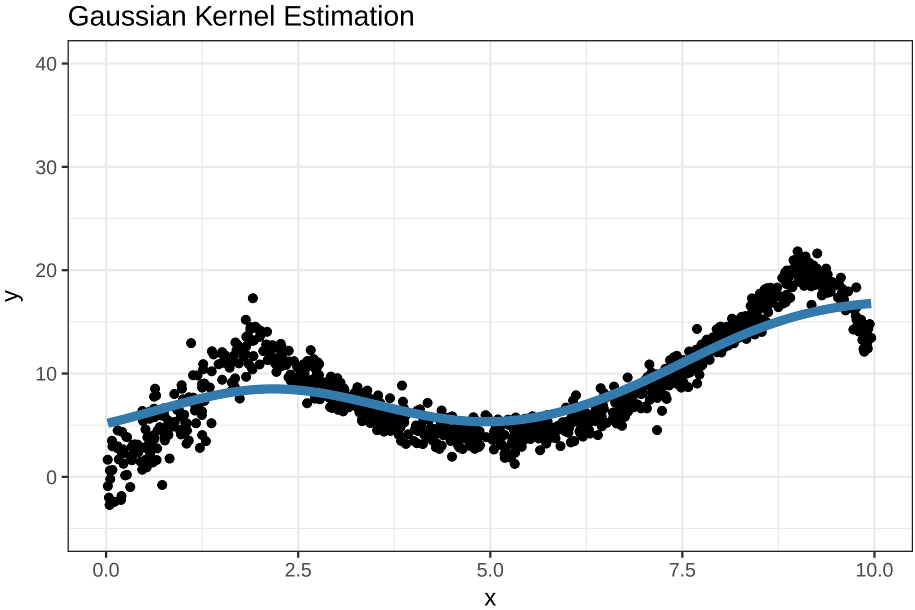

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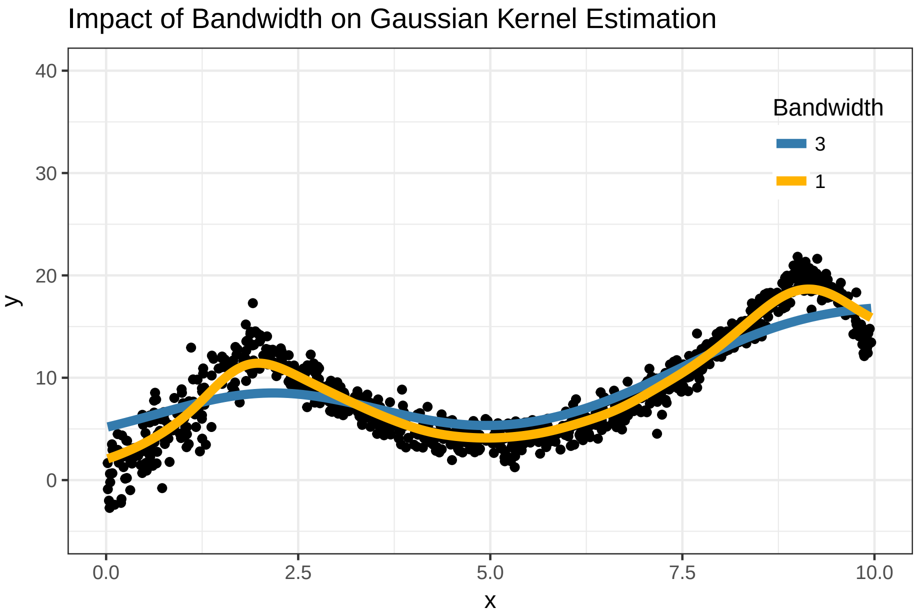

Bias-Variance Tradeoff

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  - A very inflexible estimator restricts the shape of the function to a particular form
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- We call this the linear regression line
- What function minimizes MSE, among this class of functional forms?

Best Linear Predictor

Theorem

For a random variable X and Y, if $V[X] > 0$, then the best linear predictor (BLP) of Y given X is $g(X) = \alpha + \beta X$ where,

$$\alpha = E[Y] - \frac{Cov[X, Y]}{V[X]} E[X]$$

$$\beta = \frac{Cov(X, Y)}{V(X)}$$

Corollary

- *The BLP is the best linear predictor of the CEF. I.e. setting $a = \alpha$ and $b = \beta$ minimizes*

$$E\left[(E[Y \mid X] - (a + bX))^2\right]$$

- *If the CEF is linear, the CEF is the BLP*

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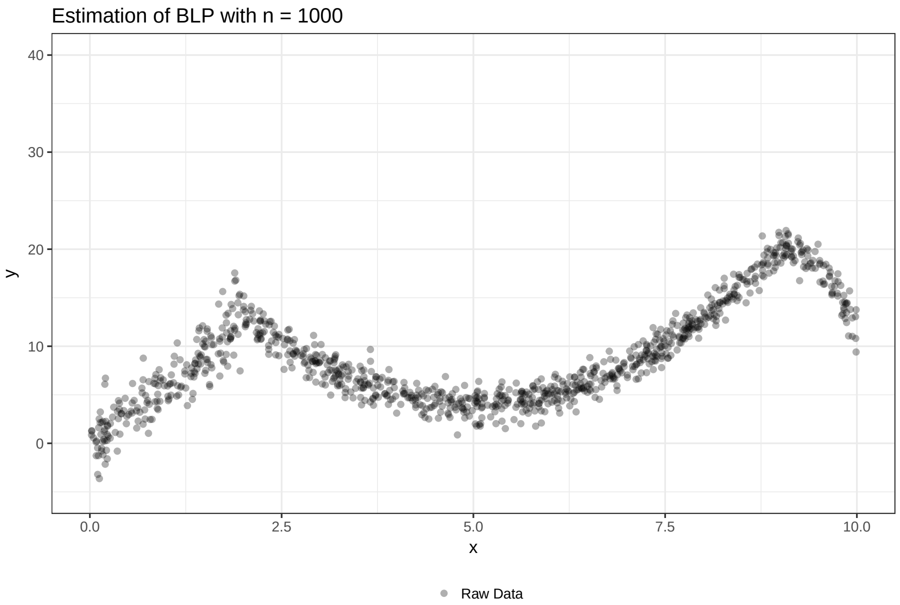

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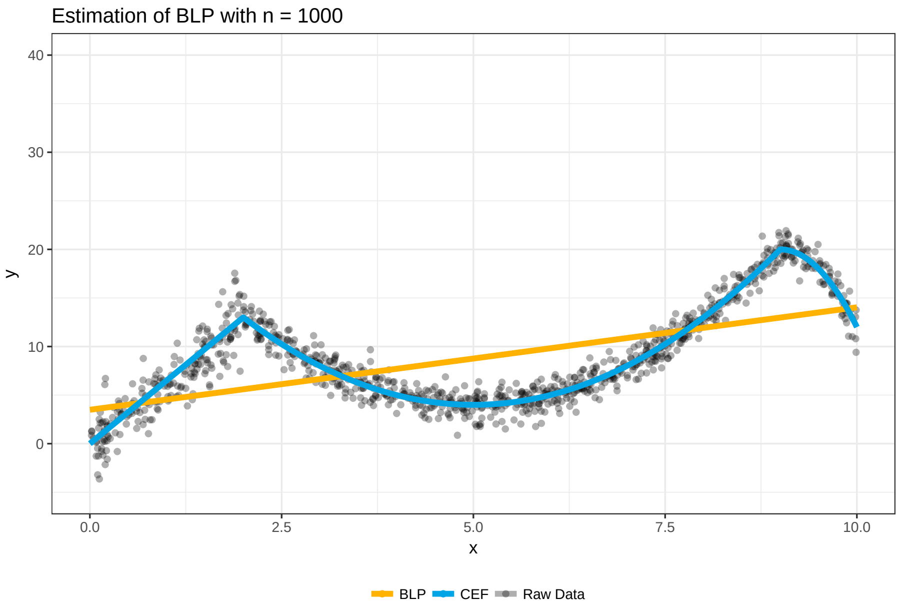

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  - Linear projection is the best predictor among linear functions.
- The nice thing about the linear projection is that it exists and is well-defined even if the CEF is non-linear.

BLP Approximations Depend on the Marginal Distribution of X

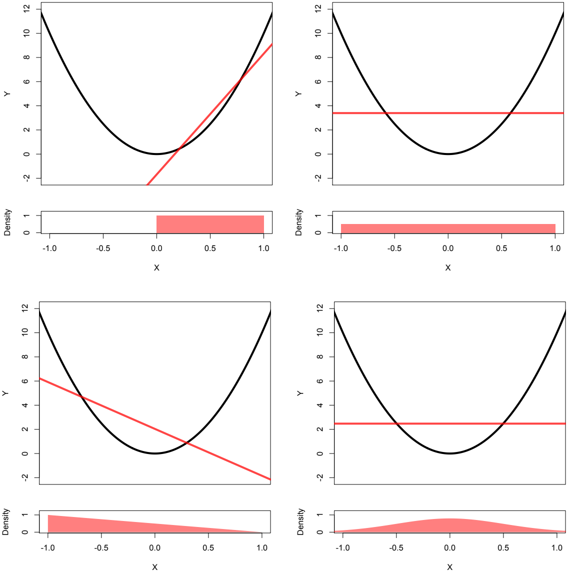

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Figure: 'If I fits, I sits'

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The BLP is always a line regardless of the data.

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- This is called the ordinary least squares (OLS) estimator.

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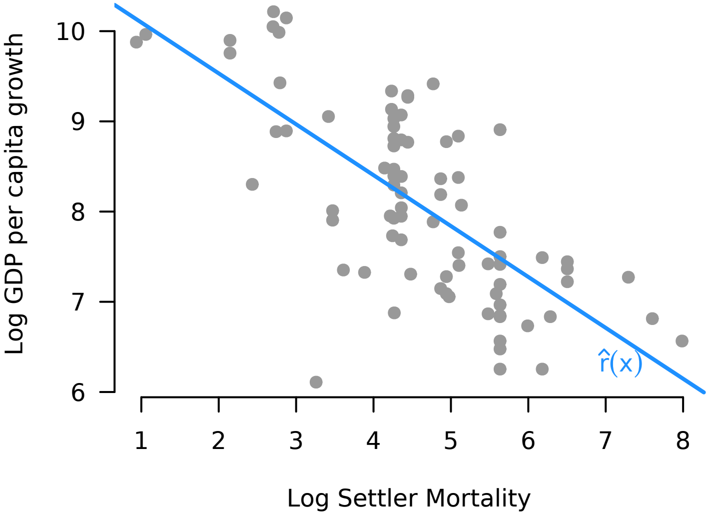
This corresponds to the linear projection which minimizes the sum of squared errors.

Fitted linear CEF/regression function



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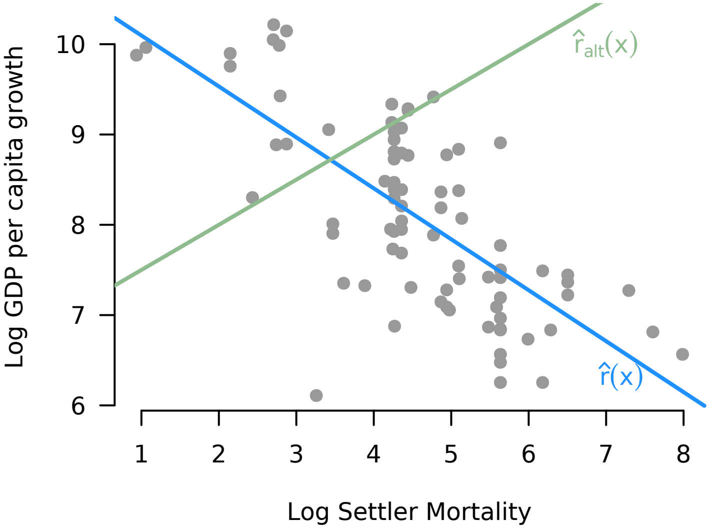

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A **fitted value** or **predicted value** is the estimated conditional mean of Y_i for a particular observation with independent variable X_i:

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Definition (Residual)

The **residual** is the difference between the actual value of Y_i and the predicted value, $\widehat{Y}_i$:

$$\widehat{u}_i = Y_i - \widehat{Y}_i = Y_i - \widehat{\beta}_0 - \widehat{\beta}_1 X_i$$

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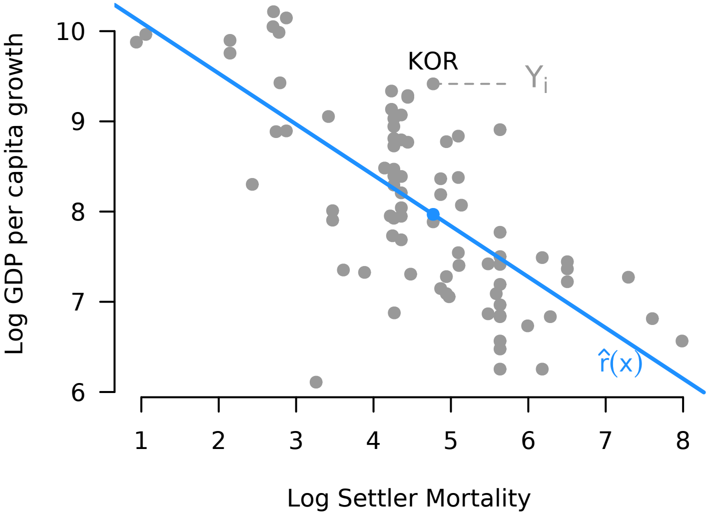

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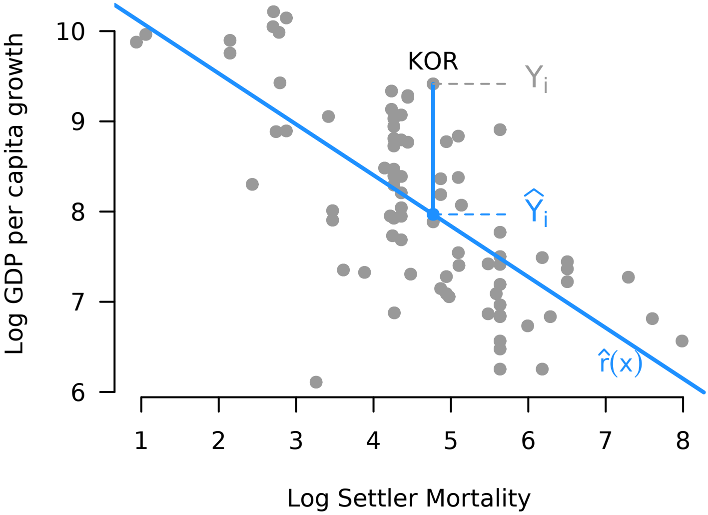

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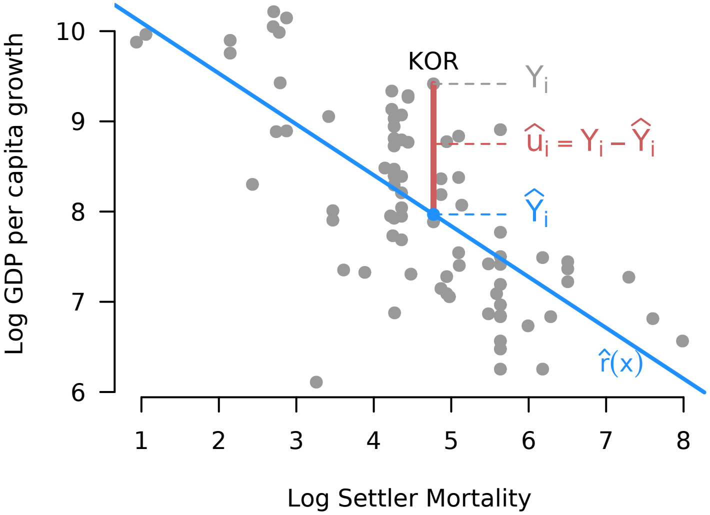
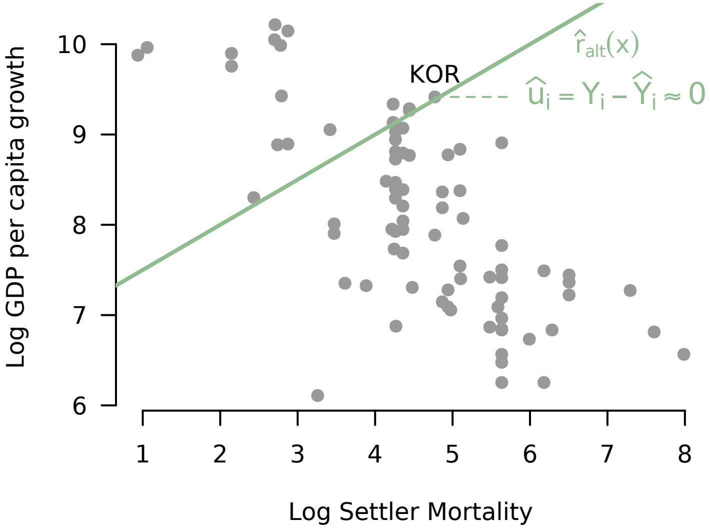


Why not this line?



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Minimize the residuals

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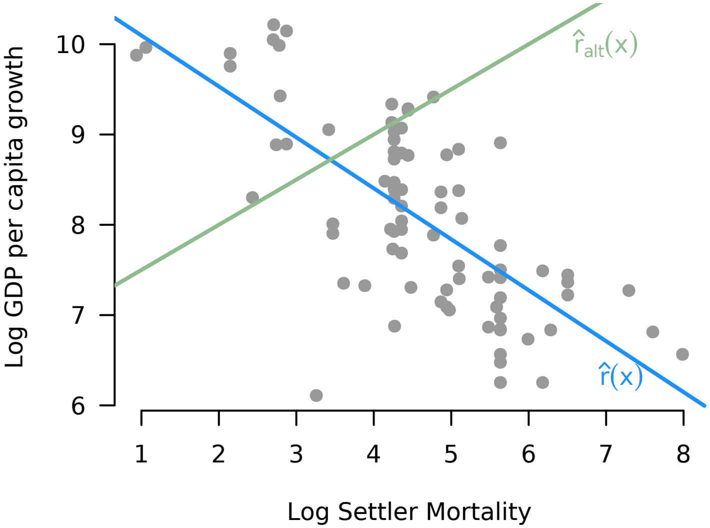
- Choose the line that minimizes the residuals

Which is better at minimizing residuals?



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- Perhaps the biggest reason is that it extends easily to the case where X is a vector of random variables.

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Next Time: Interpreting Regression

Where We've Been and Where We're Going...

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- Last Week
  - inference and estimator properties
  - point estimates, confidence intervals
- This Week
  - hypothesis testing
  - what is regression?
  - nonparametric and linear regression
- Next Week
  - inference for simple regression
  - properties of ordinary least squares
- Long Run
  - probability → inference → regression → causal inference

1 Hypothesis Testing
- Terminology and Procedure
- One-Sided Tests
- Connections
- Power

2 p-values
- Mechanics
- Multiple Testing
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- This helps clear that it is an approximation to the CEF and that the units being described are different.

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- While the line is defined over all regions of the data we may be concerned about:
  - interpolation
  - extrapolation
  - predicting in ranges of X with sparse data

Which Predictions Do You Trust?

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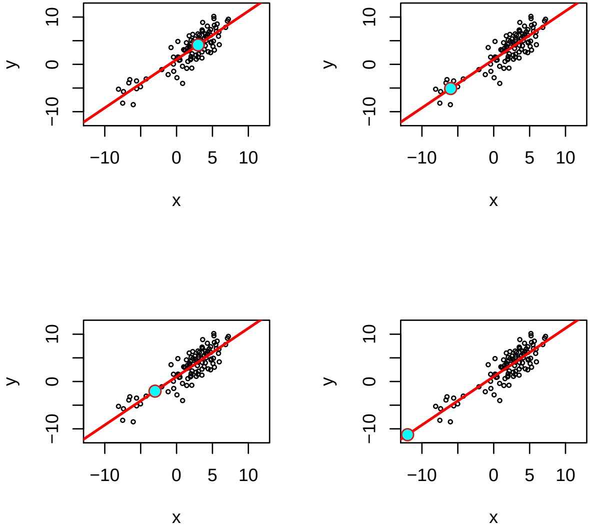

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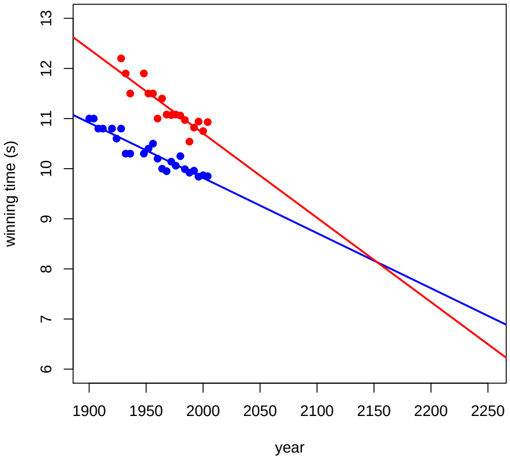
How do the authors make this conclusion?

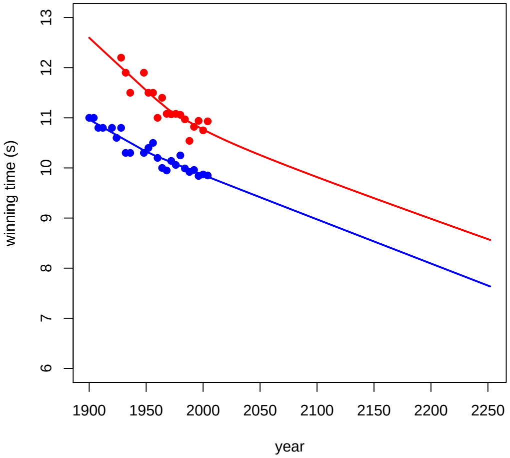
Using data from 1900 to 2004, they fit linear regression models of the winning 100 meter time on year for both men and women. They then use the estimates from these models to extrapolate 152 years into the future.

Tatem et al. Extrapolation



Tatem et al.'s predictions. Men's times are in blue, women's times are in red.

Alternate Models Fit Well, Yield Different Predictions



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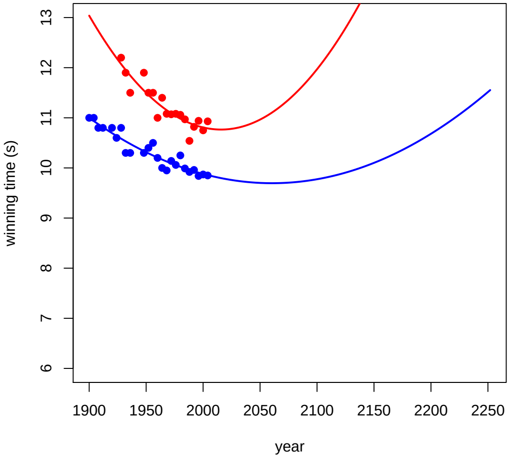

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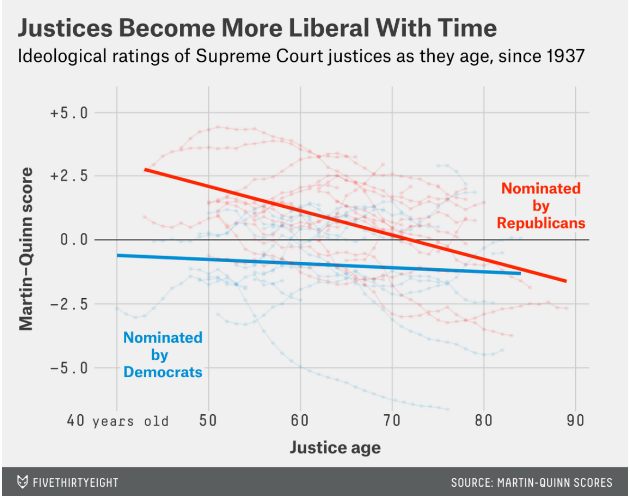

A More Subtle Example

Nate Silver @NateSilver538 · Oct 5
So, basically, John Roberts is going to be Ruth Bader Ginsburg by 2036.
53eig.ht/1Gsl2u6

Supreme Court Justices Get More Liberal As They Get Older
The Supreme Court justices are back from vacation. They've picked up their robes from the cleaners — Alito's had a pesky mustard stain — and are ...

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- For now, it is safest to treat β as a purely descriptive/predictive quantity

Fun with Linearity

"The Siren's Song of Linearity"

Fun with Linearity

Psychonomic Bulletin & Review
2007, 14 (2), 288-294

Iterated learning: Intergenerational knowledge transmission reveals inductive biases

MICHAEL L. KALISH
University of Louisiana, Lafayette, Louisiana

THOMAS L. GRIFFITHS
University of California, Berkeley, California

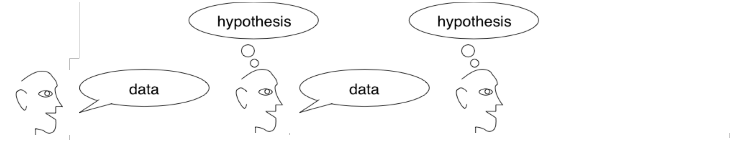
AND

STEPHAN LEWANDOWSKY
University of Western Australia, Perth, Australia

Cultural transmission of information plays a central role in shaping human knowledge. Some of the most complex knowledge that people acquire, such as languages or cultural norms, can only be learned from other people, who themselves learned from previous generations. The prevalence of this process of "iterated learning" as a mode of cultural transmission raises the question of how it affects the information being transmitted. Analyses of iterated learning utilizing the assumption that the learners are Bayesian agents predict that this process should converge to an equilibrium that reflects the inductive biases of the learners. An experiment in iterated function learning with human participants confirmed this prediction, providing insight into the consequences of intergenerational knowledge transmission and a method for discovering the inductive biases that guide human inferences.

Images on following slides courtesy of Tom Griffiths

Fun with Linearity



The Design

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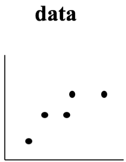

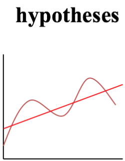

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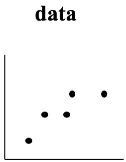

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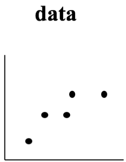

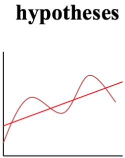

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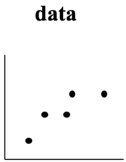

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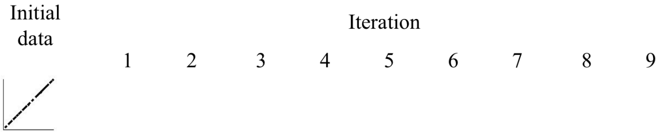
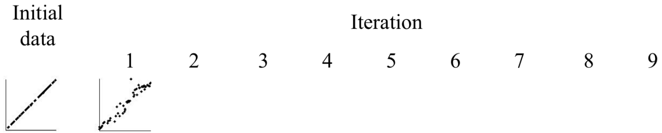
- Makes predictions of y for new x values
- Predictions are data for the next learner

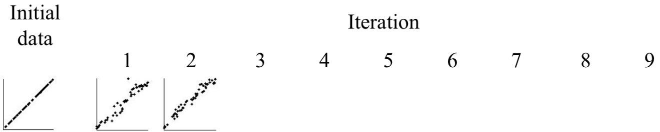
Results



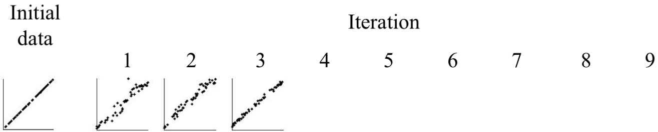
Results



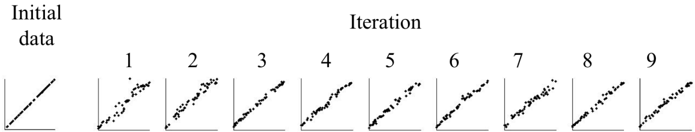
Results



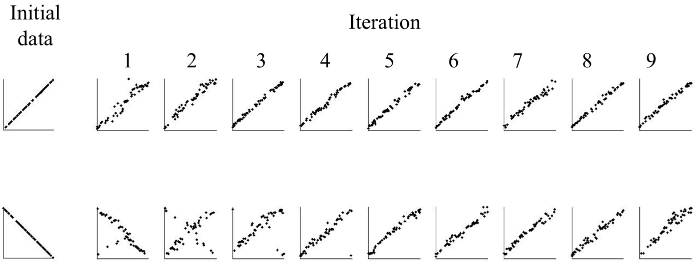
Results



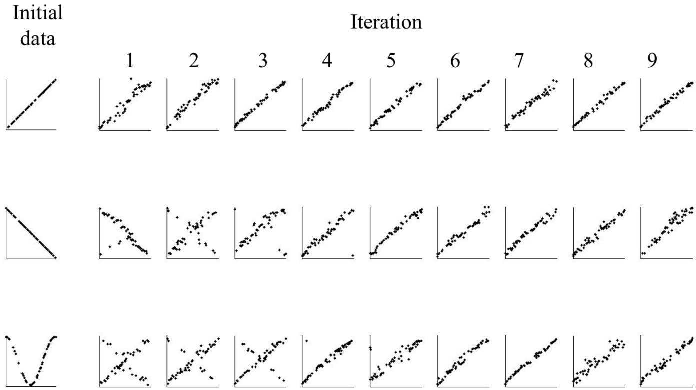
Results



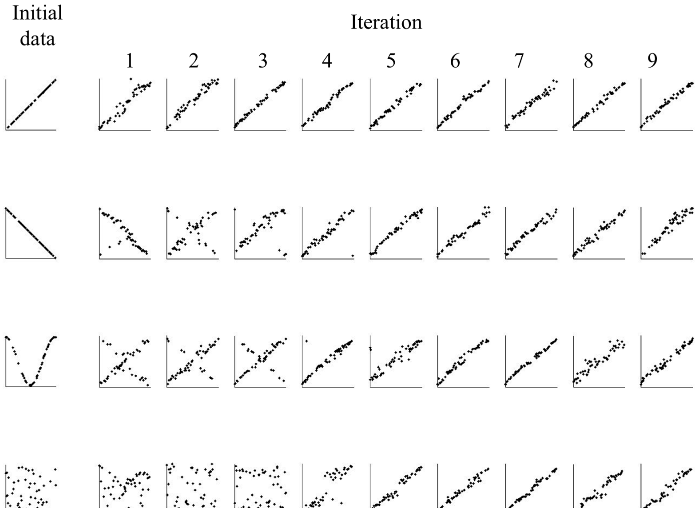
Results



Results



Results



We covered

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- Some basic insights about how to interpret regression.
- Issues of extrapolation.
- We will return to this more in future weeks.

This Week in Review

- Hypothesis Testing!
- P-Values!
- What is Regression?
- Interpreting Regression!

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Next week: Properties of Linear Regression with One Explanatory Variable.