Soc500: Applied Social Statistics
Week 1: Introduction and Probability

Brandon Stewart[1]

Princeton

September 14, 2016

[1] These slides are heavily influenced by Matt Blackwell, Adam Glynn and Matt Salganik. The spam filter segment is adapted from Justin Grimmer and Dan Jurafsky. Illustrations by Shay O'Brien.

Where We've Been and Where We're Going...

- Last Week
  - methods camp
  - pre-grad school life
- This Week
  - Wednesday
    - welcome
    - basics of probability
- Next Week
  - random variables
  - joint distributions
- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression

Questions?

Welcome and Introductions

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- When we are done you will be able to teach yourself many things

Specific Goals

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  - feel empowered working with data

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  - build a solid, reproducible research pipeline to go from raw data to final paper
  - provide you with the tools to produce your own research (e.g. second year empirical paper).

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Why RMarkdown?

What you've done before

Baumer et al (2014)

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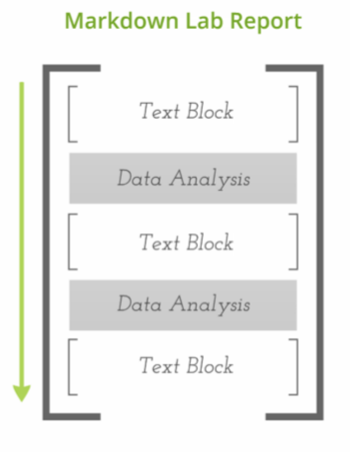

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- Crucially though- this class is not about statistical aptitude, it is about effort

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Note: You may find these difficult. Start early and seek help!

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Your Job: get help when you need it!

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- Also thanks to those who have discussed with me at length including Dalton Conley, Chad Hazlett, Gary King, Kosuke Imai, Matt Salganik and Teppei Yamamoto.
- Shay O'Brien produced the hand-drawn illustrations used throughout.

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  3. statisticians generalize and export the best of these methods

Quantitative Research in Theory

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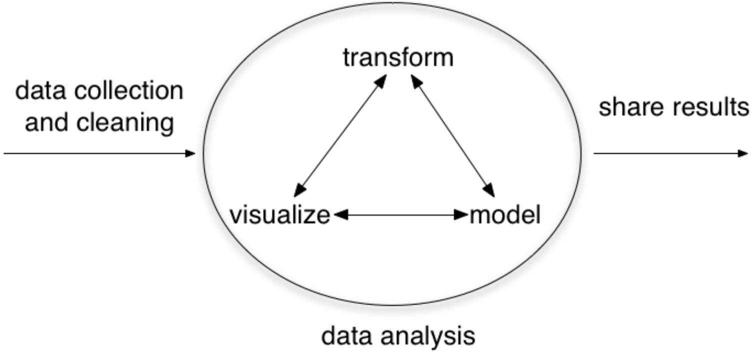

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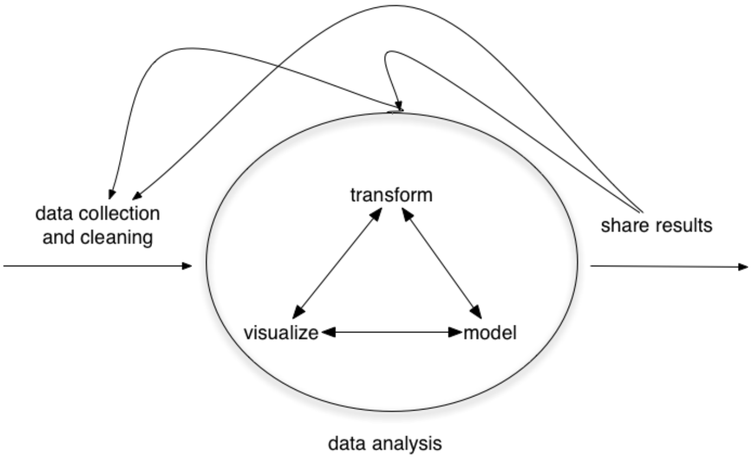

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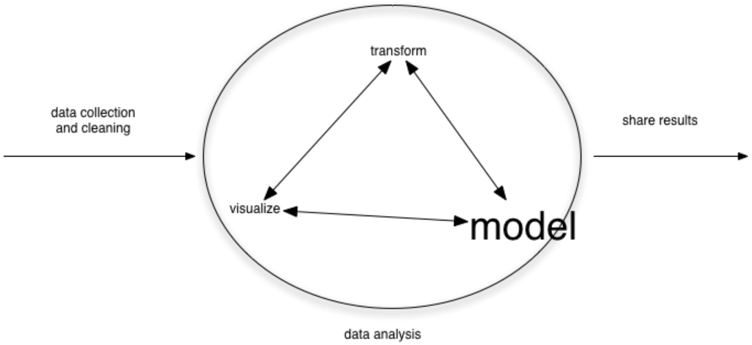

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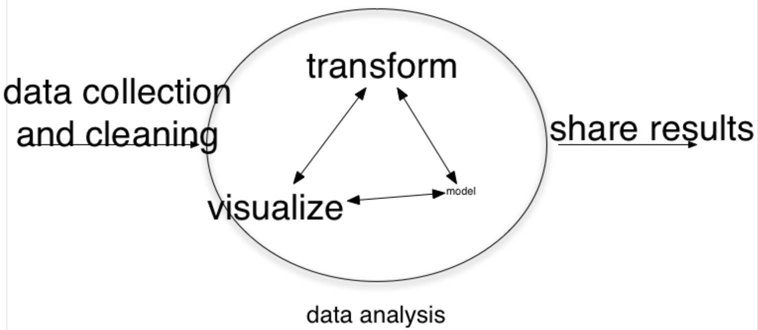

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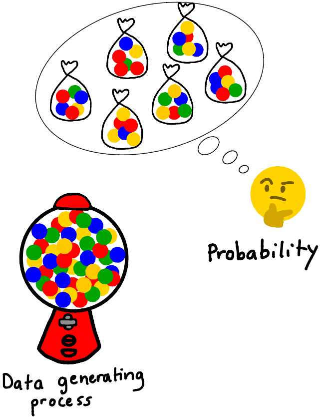
- Our way of talking about stochastic outcomes is probability.

In Picture Form



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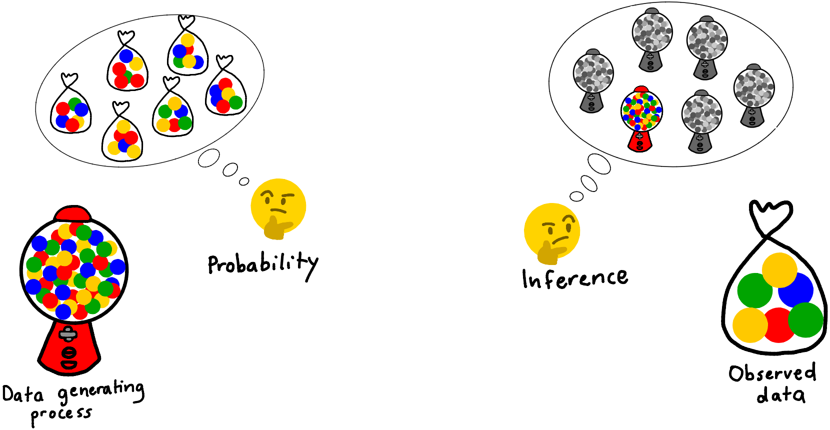

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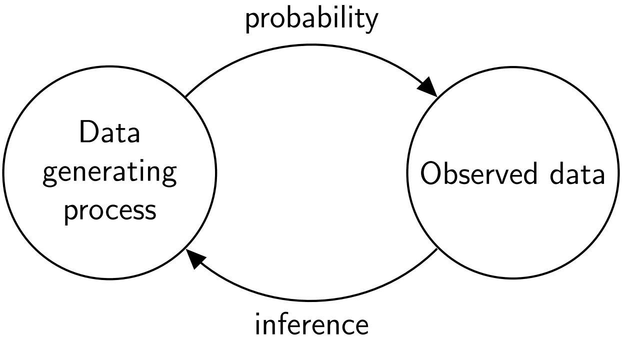

Statistical Thought Experiments

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- We will review probability today, but feel free to ask questions as needed.

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Tea-Tasting Distribution

Success count	Permutations of selection	Number of permutations
0	oooo	1 × 1 = 1
1	ooox, ooxo, oxoo, xooo	4 × 4 = 16
2	ooxx, oxox, oxxo, xoxo, xxoo, xoox	6 × 6 = 36
3	oxxx, xoxx, xxox, xxxo	4 × 4 = 16
4	xxxx	1 × 1 = 1
Total		70

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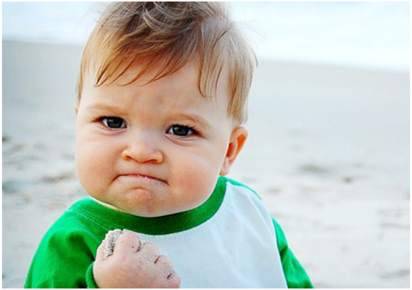

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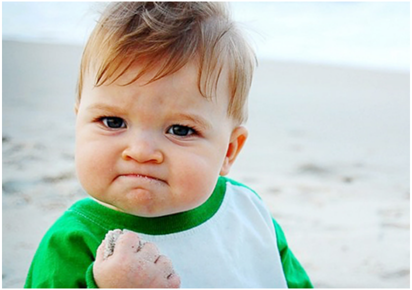

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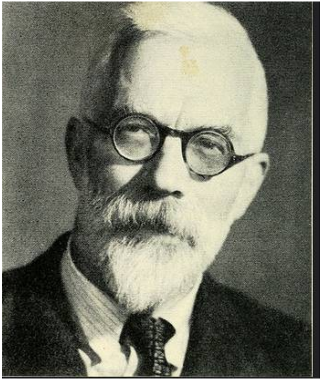

This became the Fisher Exact Test.

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- Thus: we need to know how probability gives rise to data

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3 Axioms (Intuitive Version):

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(Note we defined illogical guesses to be prob$= 0$)

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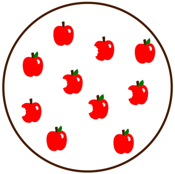

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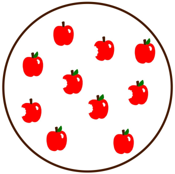

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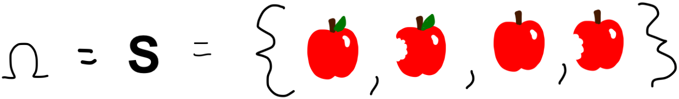

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One way to define an event is to describe the common property that all of the outcomes share. We write this as

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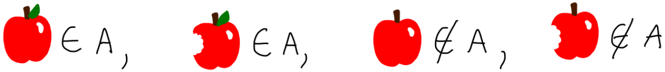
If $A = \{\omega | \omega \text{ has a leaf}\}$:



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A complement of event A is a set: A^c, is collection of all of the outcomes not in A. That is, it is "everything else" in the sample space.

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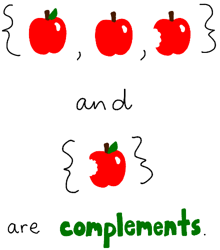

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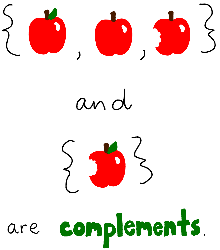

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Important complement: $\Omega^c = \emptyset$, where $\emptyset$ is the empty set—it's just the event that nothing happens.

Operations on Events

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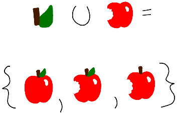
The union of two events, A and B is the event that A or B occurs:



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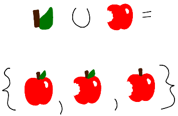

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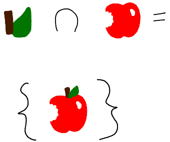

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Sample spaces can have infinite outcomes $A_1, A_2, \ldots$.

Probability Function

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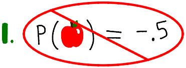

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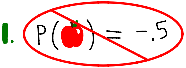

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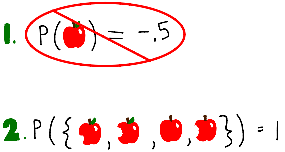

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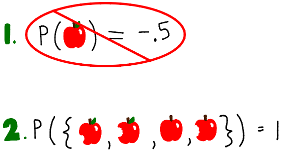

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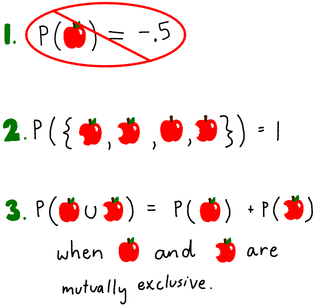

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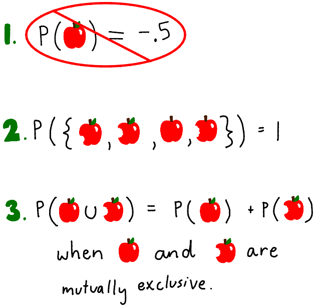

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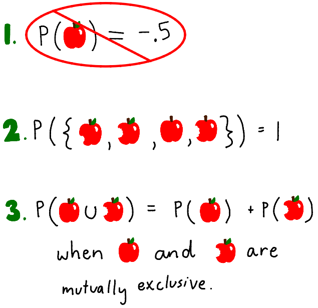

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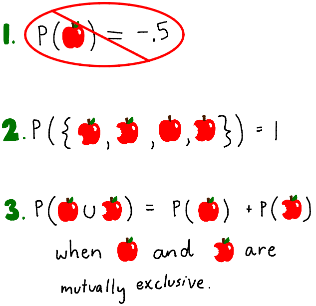

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Intuition: probability as allocating chunks of a unit-long stick.

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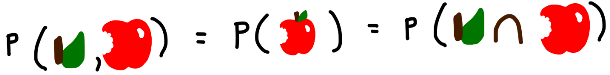
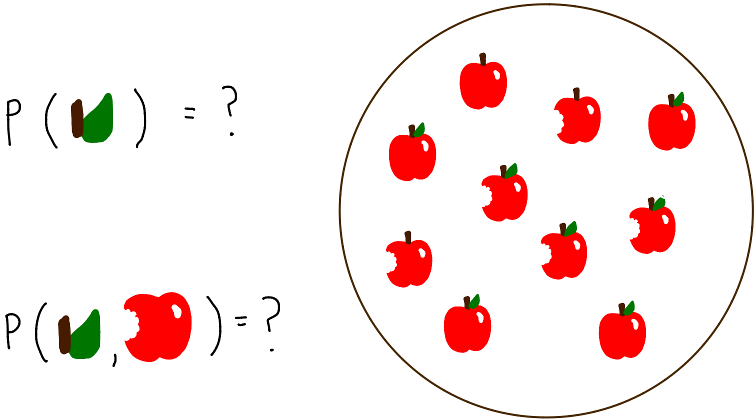
Suppose we are now in a situation where we would like to express the probability that an event A and an event B occur. This quantity is written as $P(A \cap B)$, $P(B \cap A)$, $P(A, B)$, or $P(B, A)$ and is the joint probability of A and B.

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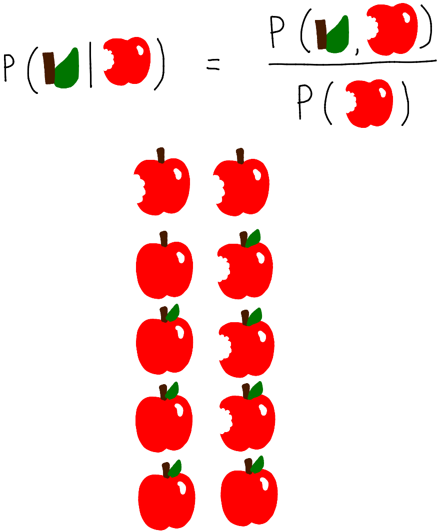
Conditional Probability: A Visual Example



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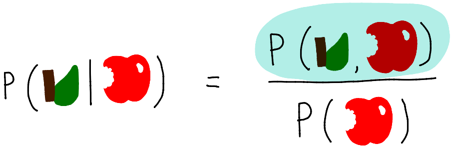

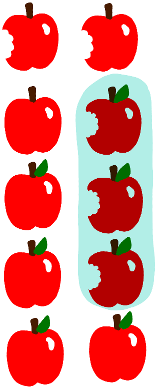

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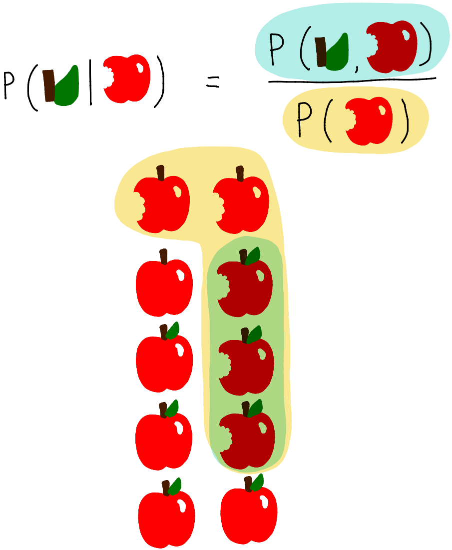

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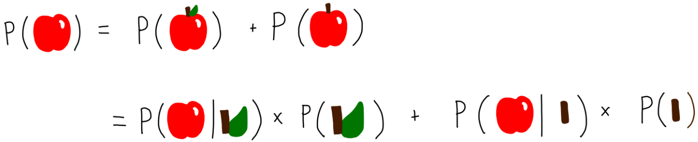
Law of Total Probability (LTP)

With 2 Events:

$$\begin{aligned} P(B) &= P(B, A) + P(B, A^c) \\ &= P(B|A) \times P(A) + P(B|A^c) \times P(A^c) \end{aligned}$$



Recall, if we randomly draw two cards from a standard 52 card deck and define the events $A = \{\text{King on Draw 1}\}$ and $B = \{\text{King on Draw 2}\}$, then

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Question: $P(B) = ?$

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$$
\begin{aligned}
P(B) &= 3/51 \times 1/13 + 4/51 \times 12/13 \\
&= \frac{3 + 48}{51 \times 13} = \frac{1}{13} = \frac{4}{52}
\end{aligned}
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\end{aligned}$$

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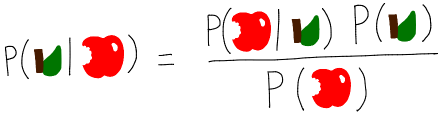
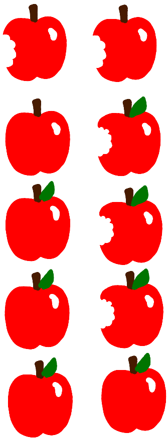
- Proof: combine the multiplication rule $\Pr(B|A)\Pr(A) = P(A \cap B)$, and the definition of conditional probability

Bayes' Rule Mechanics





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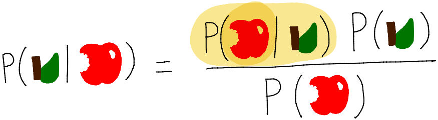

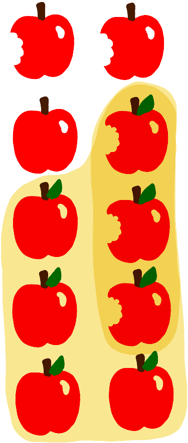

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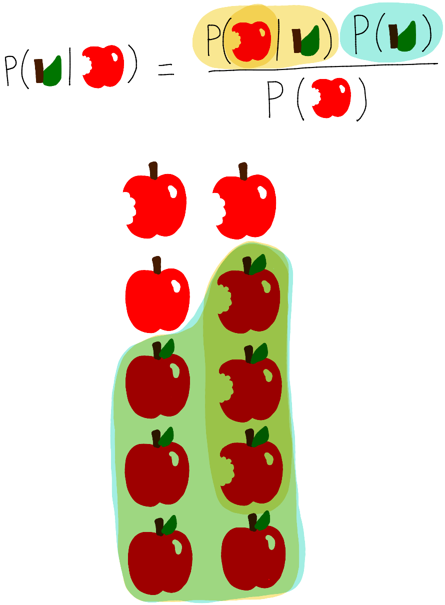

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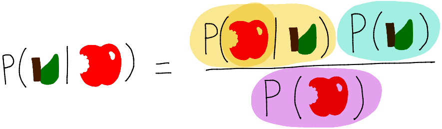
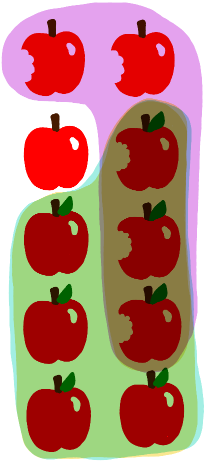
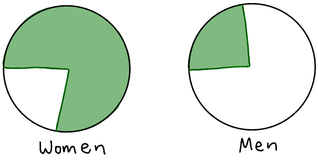




Bayes' Rule Example

U.S. Billionaires, 2014



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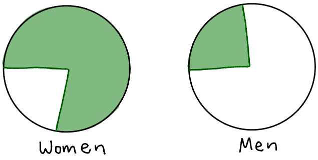

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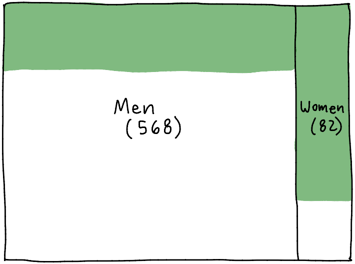
$$P(W \mid I) = \frac{P(I \mid W)\, P(W)}{P(I)}$$

$$= \frac{.765\left(\frac{82}{568+82}\right)}{\frac{.765(82) + .245(568)}{568+82}}$$

$$= .31$$



* Data source = Billionaires characteristics database

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$$\Pr(\text{AfAm}|\text{Wash}) = \frac{\Pr(\text{Wash}|\text{AfAm})\Pr(\text{AfAm})}{\Pr(\text{Wash})}$$

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Independence is a massively important concept in statistics.

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- A word from your preceptors

Fun With

Fun with

Fun with History

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Legendre

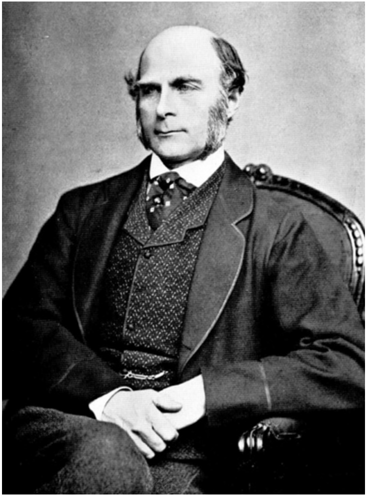
Fun with History

Gauss

Fun with History

Quetelet

Fun with History



Galton

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